M2T2

HOW MUCH SPACE DO YOU NEED?

Measurement

STATE GOAL 7: Estimate, make and use measurements of objects, quantities and relationships and determine acceptable levels of accuracy

Statement of Purpose:

The Illinois Learning Standards state that measurement provides a way to answer questions about "how many", "how much", and "how far." During the middle school years it is important that we help students to make connections between different systems of measurement (metric and traditional), different units of measurement (feet vs. inches, cm vs. m), and different methods of measurement (direct measurement, comparison, estimation, use of an appropriate instrument). It is during this time that students should also begin to understand the relationships between measurements in one, two and three dimensions (length, area and volume). As the NCTM *Principles and Standards* points out, measurement skills and concepts can be developed across the curriculum and throughout the year. With the hands-on activities in this unit we strive to build the students' understanding of the concept of volume and have them use this understanding to build a model.

Highlights

In this unit, the main project asks the students to design and construct an aquarium for a classroom. They will explore volume and how it is determined by the shape of the container. They will work with estimation, unit conversion, space constraints, and net diagrams.

Connections to the Illinois Learning Standards.

Standard 7.A.—Measure and compare quantities using appropriate units, instruments, and methods. Participants develop the concept of volume by building rectangular prisms with cubes and by folding nets.

Standard 7.B.—Estimate measurements and determine acceptable levels of accuracy. Participants measure capacity and produce a geometric model of an aquarium using appropriate measurements. The capacity of the resulting aquarium must meet a specified level of accuracy.

Standard 7.C.—Select and use appropriate technology, instruments, and formulas to solve problems, interpret results, and communicate findings. Students work together to design and build the fish tank model. They must answer questions and explain mathematical aspects of their design, such as the formula for the volume.

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Note: Appendices are printed only on the odd pages. This is done to make photocopying easier. That is, each participant should have a copy of all the odd numbered pages. While the instructors should have a copy of all the pages.

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Materials

Minimal:

- Several assorted containers; some with capacity of exactly one gallon and others of various sizes
- Filler material for checking capacity. Some suggestions: Rice, rock salt, dry sand, small beans or lentils.
- One-inch cubes (100 - 200 per group of 3-4 participants)
- Calculators
- Newsprint
- Rulers and yardsticks or measuring tapes
- Protractors
- Scissors
- Poster board or large pieces of cardboard
- Tape

Optimal list includes:

- Clear plastic geometric volume set
- Internet access
- Art supplies for finishing up fish tanks
- Computer clip art

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One guideline for the size of the aquarium is *1 gallon for every inch of fish in the tank.*

1 gallon is approximately equal to *231 in^3*

How Big Is a Gallon?

To begin this project the teacher announces that the class will be getting a fish and needs to design an aquarium for them (*or* they have been asked to design a tank for another class who are getting the fish). The fish will be 3 inches long. Through classroom discussion or research it should be determined that fish tank sizes are usually given in gallons. It makes sense then to begin the project with a discussion of what a gallon looks like.

Step-by-step guide

ACTIVITY 1

⇒ The teacher first shows the students several familiar gallon containers; ice cream cartons in different shapes, milk jugs, large zip lock bag, etc. One is filled with packing material, which is then poured into the others to show they have the same capacity.

⇒ The teacher then brings out several different shaped containers, cereal box, can, platter, bowl, etc. and asks the students to indicate whether they think the shape will hold less than a gallon, a gallon, or more than a gallon. *See worksheet at right.*

⇒ After all students have compared each container to a gallon container, the teacher then uses the filler material check the capacity of each container.

ACTIVITY 2

⇒ Arrange students in groups of four and give each group an assortment of five containers (margarine tub, soup can, tuna can, spice jar, etc.)

⇒ Each group cooperatively arranges the containers from least to greatest capacity by estimating.

⇒ When the group agrees, the order is recorded.

⇒ Next the group uses the packing material to check their estimates and to make any necessary adjustments.

Discussion of Math Content and Related Questions

- Discuss the attributes that affect capacity. How can so many different containers all have a volume of one gallon?
- Which containers created the most discussion in the small group activity? Why? Discuss the relationships between volumes and measurements of length, width, height, area, perimeter, radius.
- If available, demonstrate the relationships between the various containers in the Clear Plastic Geometric Volume Set

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Participant Page

Activity 1 - How Big is a Gallon?

Name____________________________

*What we are comparing here is the **capacity** of these containers.*

Some suggested materials for checking capacity:
-rice
-rock salt
-dry sand
-small beans or lentils

For each container shown, check the appropriate column for your estimate.

Container	Holds less	Holds exactly a	Holds more	
1				
2				
3				
4				
5				
6				

Activity 2 - Which Contains the Most?

Estimates	
Least capacity	
Greatest	

Measurements	
Least capacity	
Greatest	

HOW DID YOU DO?

As the shapes are filled, check the column if you were correct.

What was your personal # of correct? ________________

What was the group # of correct? ________________

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Instructor Page

During this activity students will naturally begin to move from the counting of cubes to an understanding of finding capacity by multiplying

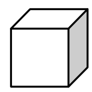
EXTENSION: Some students may find conversion factors of gallons to cubic feet, or cubic yards, etc. If time permits, the classroom discussion of how to convert to cubic inches would be a great extension of this project.

How Length, Width, and Height Affect Volume

Continue the discussion of the problem of designing an aquarium. One problem is that fish tanks are typically sized in gallons, but their linear measurements are usually in inches. In designing our tanks we will be working in inches. We need to develop a method for determining capacity in cubic inches. We can research a conversion table and find that one gallon is approximately equal to 230 cubic inches. Participants will work with actual one-inch cubes to help visualize this.

After a brief introduction and demonstration by the instructor, this activity is designed to allow the students to work at their own pace. They need to explore with the cubes and take time to make connections between changes in length, width, height and capacity.

Step-by-step guide



- ⇒ Start with one cube. It has a length of 1 inch, a width of 1 inch, and a height of 1 inch. Its volume is 1 cubic inch. We will call it a one-inch cube.
- ⇒ Use one-inch cubes to build two or three rectangular prisms of height one inch. Demonstrate that the volume of each is the number of cubes necessary to build it.
- ⇒ Use more cubes to add additional layers to these rectangular prisms to increase their heights. Again the volume of each prism is the number of cubes necessary to build it.
- ⇒ Students continue to build their own rectangular prisms with their groups, recording the length, width, height, and volume of each.

Discussion of Math Content and Related Questions

- For prisms (and cylinders) of the same height, the figure with the largest base area has the largest Volume.
- For prisms (and cylinders) of the same base area, the solid with the greatest height has the greatest Volume.
- Often the solution to a small problem can be determined by counting or measuring (just as we have been counting the cubes to determine volume). Mathematics becomes necessary when the problem becomes too large. Can we find a mathematical process for finding Volume? Can we express the process symbolically as a formula?

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Participant Page

Make sure you label all your answers correctly with inches, square inches or cubic inches.

Length, width and height are all important to Volume. Pay attention to how each is changing.

How Length, Width, and Height Affect Volume

- Use one inch-cubes to build a rectangular prism.
- Count the cubes to find the length and width of the base rectangle and record in the table below.
- Record the Base area.
- Count the layers of cubes to find the height of the prism.
- Record the total number of cubes in the column for Volume.
- Continue to build more rectangular prisms and record this information for each.

Length (inches)	***Width (inches)***	***Base Area***	***Height (inches)***	***Volume***

What is happening to the Volume of the prisms as the other measures change?

Can you find a rule for calculating the Volume so that you don't have to count the cubes?

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Using One-inch Cubes to Build Rectangular Prisms

<u>ISAT Connection:</u>
Throughout these activities the students are asked to write about their process and understanding of the concepts. The ISAT extended response questions ask students to "explain in words how you got your answer, and why you did the steps you did to solve the problem."

The main idea on these next two pages of the worksheet is to fully move from the counting of cubes to an understanding of finding capacity by multiplying.

<u>Step-by-step guide</u>

- ⇒ Participants build the prisms shown on the worksheet and count the one-inch cubes to find length, width, height and volume.
- ⇒ Participants become able to count the cubes by visualizing the prism from its two-dimensional drawing.
- ⇒ Participants determine a "short-cut" or formula for finding volume.

<u>Possible Problems and Concerns</u>

You should move among the groups as they work on putting the information in tables to see that they are making the connection that adding a cube changes the volume by one cubic unit, while adding a row increases the volume by the number of cubes in the row. You should encourage participants to work with the blocks and not just the numbers on the paper. Try to encourage them to see that multiplication describes what is happening here.

When a group seems to have the concept down, ask them some leading questions, for example: "If my shape had one row with a length of 8 inches and I added three more rows to it, how many cubes would there now be?" (32 inch cubes) "By how much did it increase?" (3 x 8 x 1 or 24 cubes) Encourage the groups to ask questions of each other and explore various shapes besides those on the paper.

Using One-inch Cubes to Build Rectangular Prisms

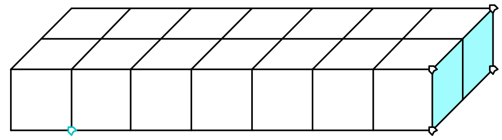
Participant Page

HINT:

Pay special attention to how many cubes you are adding each time you add a row.

Are you labeling your answers? Use inches for the length, width, and height. The volume is measured in cubic inches.

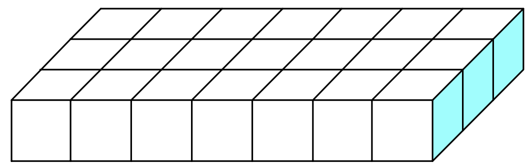
Use one-inch cubes to build this rectangular prism.



What is the length ? _____ the width? _______ the height? _________

What is the total number of cubes in the shape? __________ This number is the **Volume** of the shape in cubic inches.

Add another row.



What is the length now? _____ the width? _______ the height? _______

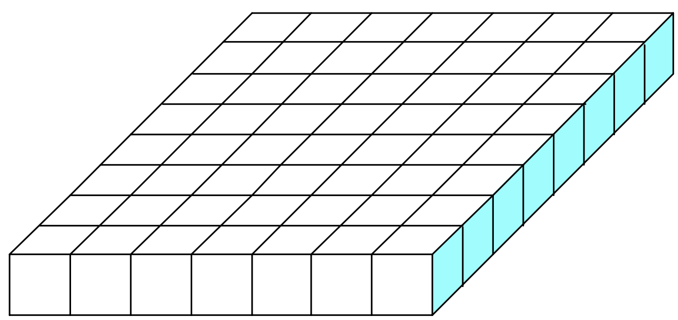
the Volume? __________

Add several more rows.

What is the length now? _____

the width? _______ the height? _________

the Volume? __________

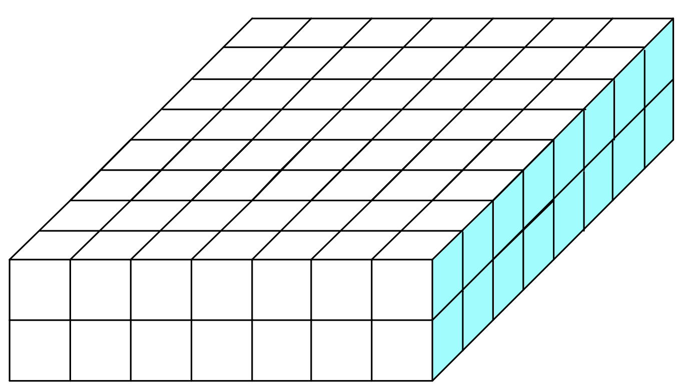


Add a whole new layer.

What is the length now? _____

the width? _______ the height? _________

the Volume? __________



Using One-inch Cubes to Build Rectangular Prisms (Continued)

Instructor Page

Students should be seeing the application of multiplication as a replacement for counting.

The Volume of a rectangular prism is often expressed as V = lwh. An equivalent, but more general, formula is V = Bh, where B is the Base area of the prism.

Many of these activities refer to volume as the number of one-inch cubes. Inches are used because gallons are typically used to size aquariums. In another application the metric system might be used. If the linear measurements are in centimeters, the volume is measured in cubic centimeters. Cubic metric units may be converted to liters.

The metric system was designed by scientists so the units were defined to be related. One cubic centimeter has capacity of one milliliter. One milliliter of water has mass of one gram.

Once again the teacher should move among the groups and see that the students are making the connections. Students may no longer need the cubes, but they should be available as a method for checking. Everyone should try to use proper vocabulary and be able to describe how multiplication can be used to find the number of cubes.

Possible Problems and Concerns

Try to listen carefully to how the students are working through the table . Are they using terms like length, width and height? Are they discussing the concepts rather than just "plugging in the formula"? Try to encourage them to talk about what the numbers represent as they are manipulating the data.

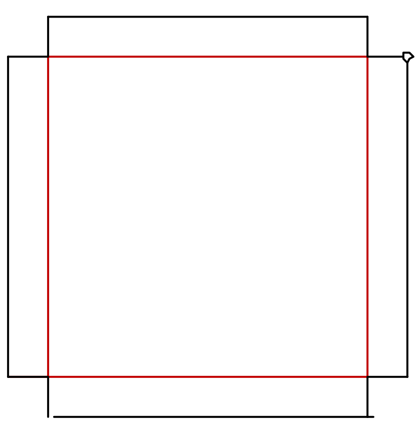
The goal is to have them understand that these numbers stand for real dimensions and not just abstract values. Knowing why they need three dimensions to find capacity is key to having them understand why they will only need two dimensions later when they compute surface area.

EXTENSION:

Box Measuring

Begin with a piece of 20-by-20 centimeter squared paper. Cut a square from one corner. Cut squares that are the same size from the three other corners and fold up what's left to make an open box. How many centimeter cubes could fit inside the box?

Make as many different size boxes as possible, using this method. Make each from a different piece of 20-by-20 centimeter squared paper.



Which of these boxes has the greatest capacity?
Explain in words how you decided which box would contain the most, and why you did the steps you did to solve the problem.

Burns, Marilyn. *About Teaching Mathematics*. p 55.

Check out the applet for this problem at www.mste.uiuc.edu/carvell/3dbox/

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Participant Page

Using One-inch Cubes to Build Rectangular Prisms (Continued)

NOTE:
The term "shortcut" on the worksheet is used in place of the more traditional term formula.

Some helpful questions for finding a short cut:

How many inch cubes are in each layer?

How do you find this number?

How many layers are there?

What would you do with these numbers to find the number of cubes?

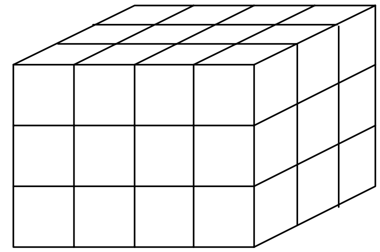
Describe the "shortcut" process in words. Then use letters (l for length, w for width, and h for height) to describe the process using symbols.

What is the length? _______

the width? _______

the height? _________

the Volume? __________

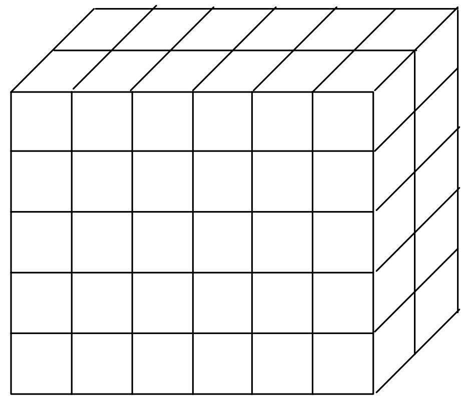


What is the length? _______

the width? _______

the height? _________

the Volume? __________

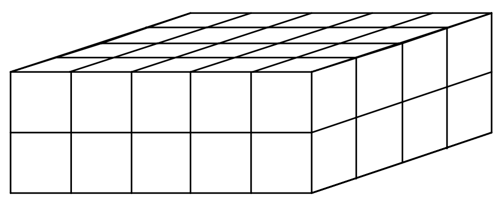


What is the length? _______

the width? _______

the height? _________

the Volume? __________



length	width	height	Volume
5	3	2	
4	4	5	
2	10	4	
11	8	7	
12	12	12	

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Changing Dimensions, Changing Capacity

The final page of this activity takes a look at what happens to the total number of cubes if we multiply one, two or all three dimensions by a constant. Initially, we are looking at a box that holds 24 one-inch cubes. As its dimensions are altered, students find the effect on the volume.

Discussion of Math Content and Related Questions

- Thinking in proportions is a powerful mathematical tool. Increasing the number of boxes by a particular scale factor increases the capacity by the same scale factor. (Doubling the number of boxes doubles the capacity. Multiplying the number of boxes by 5 makes the volume 5 times as large.)
- If two prisms (or cylinders) have the same base area, their volumes are proportional to their heights.
- When two dimensions are increased by the same scale factor, the volume is increased by the square of the scale factor. For example, if the length and width are both multiplied by three, the volume is nine times as large.
- When all three dimensions are increased by the same scale factor, the volume is increased by the cube of the scale factor. For example, if the length, width and height are all multiplied by four, the volume is 64 times as large.

EXTENSION: Have participants fill in this table as well as the one on the other page.

If the height is:	**& the length is:**	**& the width is:**	**the # of cubes will be:**
doubled	quadrupled		16 times as large
tripled	doubled	tripled	
quadrupled	quadrupled		32 times as large
doubled	unchanged	tripled	
unchanged	tripled		9 times as large
quadrupled		quadrupled	16 times as large
tripled	quadrupled		12 times as large

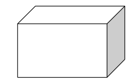
Internet Resource

The NCTM Standards Website has a wonderful, interactive example: Learning about Length, Perimeter, Area, and Volume of Similar Objects by Using Interactive Figures: Side Length, Volume, and Surface Area of Similar Solids. It is available at
http://www.standards.nctm.org/document/eexamples/chap6/6.3/part2.htm

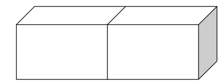
Participant Page

Changing Dimensions, Changing Capacity

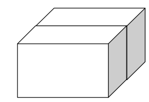
Let's assume we know 24 one-inch cubes will fit in this box.



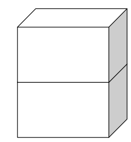
Suppose we doubled its length. How many cubes would fit in this new box ?
____



How many cubes would fit if we doubled the width?______________________

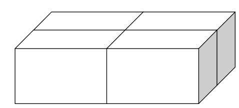


How about if we double the height?______________________________

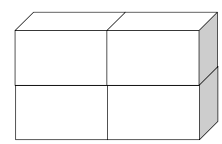


Write a statement about what you see happening______________________
____

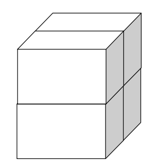
What would happen to the number of cubes if we doubled two dimensions, say the length and the width?______________________________________



Or the length and the height?______________________________

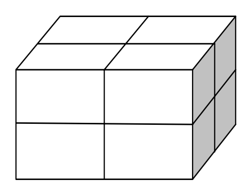


Or the width and the height?______________________________



Write a statement about what you see happening. ____________________
____

Suppose we double all three dimensions. Haw many one-inch cubes would this box now contain? ___________Explain your answer______________________
____



Think about the original box containing 24 one-inch cubes as you fill in the table.
On a separate sheet of paper write an explanation of what is happening.

If the height is:	& the length is:	& the width is:	the # of cubes will be:
Tripled	Tripled	Tripled	_______times as big
Doubled	Tripled	Unchanged	_______times as big
Quadrupled	Doubled	Halved	_______times as big

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Planning the Fish Tank

HINTS:
Give as little assistance as possible to encourage the students to work with their group to come up with a method for finding dimensions for a fish tank that has volume of 4 gallons or 924 cubic inches.

You may want the students to first sketch a scale drawing of their net diagram.

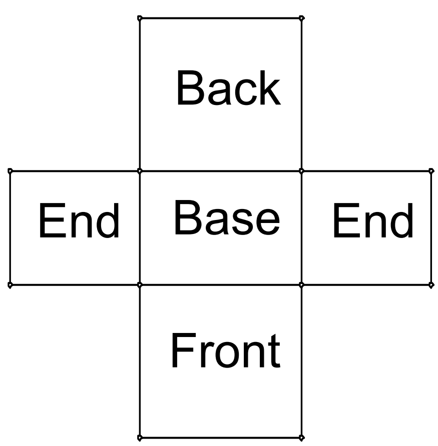
In this part of the activity, participants will use their experiences with capacity to design and build their own fish tank models. The teacher needs to be available to answer questions and clear up confusion, but for the most part needs to stay back and let the students work.

The instructor needs to decide whether to put constraints such as "whole units only" or "only dimensions that make sense in the context of a real fish tank." This will depend on how much time you can spend on the activity and how many optional topics you want to discuss.

You could talk about the infinite number of combinations of dimensions for rectangular prisms, if fractional units are allowed. If fish tanks with the same height are compared, which designs have the largest capacity? What strategies could be used to maximize capacity? In what other situations is maximizing capacity of interest?

Optional Continuation of Problem:

We now know how to determine the capacity of our fish tank. We know that the fish will need 4 gallons of water, and each gallon takes approximately 231 cubic inches. Next, each group of students needs to determine the design of their fish tank. The tank has to sit on a shelf that is 12 inches wide, so it can be no more than 10 inches wide. Point out that an actual fish tank will need to be larger than the 4 gallons calculated, because there should always be some extra space at the top so the fish do not jump out. Use the worksheet on the opposite page to keep track of the possible shapes and sizes you find.



Sketch a net of the design that your group chooses from the worksheet list of possibilities.

This net is for an aquarium shaped like a rectangular prism.

***Videa Resource*:**

"Math in the Middle….of Oceans " includes a video segment and print materials devoted to Aquariums. It shows a variety of mathematical aspects of setting up and maintaining an aquarium.
www.mathinthemiddle.org/

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Planning Fish Tank

WHAT SIZE TANK WOULD YOU LIKE?

In the table below list as many combinations of dimensions as you can for your aquarium design. Remember that the width can be no more than 10 in. and each volume should equal at least 924 in^3. (There are $231 in^3/gal$.)

Shape of the Base	Dimensions of the Base (inches)	Area of the Base (square inches)	Water Height (inches)	Height of the tank (inches)	Volume (cubic inches)	Name of this geometric solid

Examine the possible aquarium designs in your chart. Decide which will make a good aquarium and sketch a net for one that you might use to build your fish tank model. Label the sketch with the dimensions. Explain why you chose this design.

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The time given over to this part of the task is up to the teacher, but two class periods are probably required for the actual building of the model.

HINTS:
You can contact local appliance stores or moving companies for large boxes to use instead of poster board.

You may want to have some parent help for the actual cutting of the diagrams.

Building the Fish Tank

Step-by-step guide

⇒ Hand out the direction sheet and rubric to the students.

⇒ Stress that they should carefully follow every instruction and examine the rubric.

⇒ Provide materials for construction of the fish tank model.

⇒ Move among the groups to assure they are staying on task.

⇒ Give periodic time checks if time is an issue. Make sure you leave time for clean-up at the end of class.

Possible Problems and Concerns

- After a group has completed the activity on the previous page and made a design decision, they will need to make a full-size net on newsprint. When placing the net on the poster board, some of the "fold" lines may need to be cut so that it fits on the allotted poster board.

- Even though the students will be enjoying the task, the teacher needs to make sure the mathematics is not lost on them. Encourage them to "measure twice and cut once". Have them double check the diagram for accuracy. Encourage them to "speak mathematically" with each other when discussing the problems that arise. Keep your sense of humor and have fun along with them.

- You will need to decide if groups who "make mistakes" can have a second piece of cardboard.
- You will need to decide when a group has fallen so far behind time-wise that you need to lend a hand.
- You will need to mediate groups that have a breakdown in communication.

Extension:

When participants have finished building their fish tanks, have them calculate the surface area as well as the volume. (The surface area will not include the opening of the fish tank or the inside.) The volumes should be very similar, but the surface areas may vary. Have them produce ratios of surface area to volume and determine which fish tanks make the most efficient use of material. That is, which give the greatest volume for the surface area of material.

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Participant Page

BUILDING THE FISH TANK

Group members ________________
____
____
____

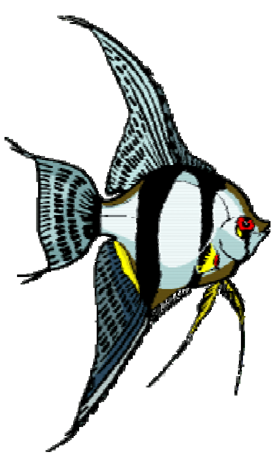
There is a very old and wise adage for carpenters that says "Measure Twice , Cut Once" If you do not heed this advice you may find out what it means!

Work with your group to design and build a model for a fish tank for the classroom fish. There are four fish that are each about three inches long. These fish need a tank that contains 4 gallons of water with a little room at the top. The aquarium will be placed on a shelf that is 12 inches wide so the tank can have a maximum width of 10 inches. The model will be constructed out of poster board and may not use more than 1 piece of 22 by 28 inch poster board. Read all of the directions and review the scoring rubric before you begin to work.

1. Determine the shape of your aquarium base and the height that will make the volume at least 4 gallons. Enter the information in the table below.

Shape of the Base	Dimensions of the Base (inches)	Area of the Base (square inches)	Water Height (inches)	Height of the tank (inches)	Volume (cubic inches)	Surface Area of the outside (square inches)	Name of this geometric solid

2. Use newsprint and draw a full-size net for your fish tank. Label the edges with the correct dimensions. Label the base and the lateral faces.
3. Position your diagram on your piece of poster board. You may have to cut on the folds of your net to make it fit on the poster board. Ask your teacher to check your diagram before you cut your poster board.
4. Cut out and assemble your fish tank. Draw a line where the water height would be.
5. Tape this instruction paper to your aquarium model.
6. Submit your tank for the capacity test and scoring.



SCORING RUBRIC

Capacity to the water line is at least 4 gallons. (25 points)	________
The width is no more than 10 inches. (5 points)	________
The construction is accurate. (10 points)	________
This paper is completed. (5 points)	________
Creativity (5 points)	________
TOTAL	________

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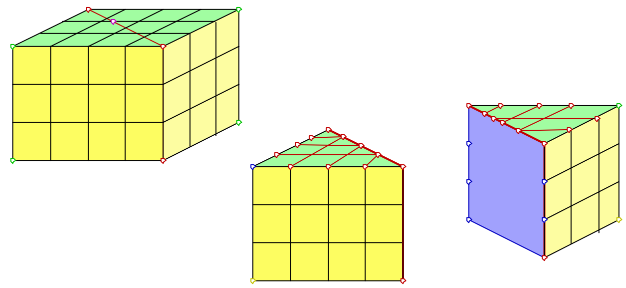
Additional Activities: Volumes of Prisms and Cylinders

The cube stacking activities focused on rectangular prisms, but the generalization that **Volume** can be found by **multiplying** the **area** of the **base** by the **height** is true for other prisms and for cylinders.

If a rectangular prism is cut in half to make two triangular prisms, the volume of each is one-half of the original rectangular prism. The volume of the prism is the area of the base triangle multiplied by the number of layers.

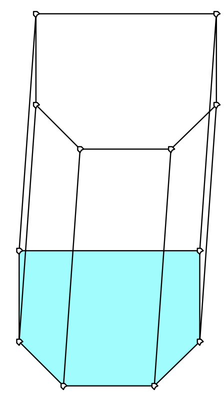


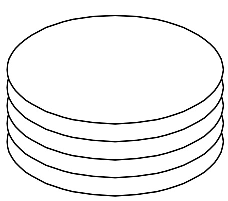
The volume of a hexagonal prism is area of the base hexagon multiplied by the number of layers. The Base area of this hexagon is 11 square centimeters. Its height is 6 centimeters. The volume is 6 layers of 11 or 66 cubic centimeters.

V = Bh

V = 11 x 6

V = 66 cm^3





A cylinder can be thought of as layers of circles or a stack of pancakes. The base is a circle. Its radius is 10 centimeters so the Base area is πr^2 or 314 square centimeters. The height is 4 centimeters. Four layers of 314 makes the volume 1264 cubic centimeters.

V = Bh

V = 314 x 4

V = 1264 cm^3

On a separate sheet of paper, draw a solid figure and find the formula for its volume..

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Additional Activities: Cube Stacking

Imagine a box that is 36 inches on every edge. This box is a cubic yard. Imagine filling this box with one-inch cubes.
How many of these cubes would be in one row of the bottom layer?________________

How many of these rows would be needed to cover the bottom of the box?___________

How many cubes would be in the bottom layer?_____________

How many of these layers are needed to fill the entire box with one-inch cubes?_______________

How many cubic inches are in the cubic yard?________________

Next imagine all of these cubes stacked one on top of the other.
How high would they tower? ___________________

Compare this stack of cubes with the Sears Tower or the Gateway Arch. Would the stack be higher than the Sears Tower?

____

Metric Boxes

Imagine a box that is one meter on every edge. This box is a cubic meter. Imagine filling this box with one-centimeter cubes. How many of these cubes would be in one row of the bottom layer?

____

How many of these rows would be needed to cover the bottom of the box? ________________

How many cubes would be in the bottom layer? ________________

How many of these layers are needed to fill the entire box with one-centimeter cubes? _______

How many cubic centimeters are in the cubic meter? ________________

How long would it take to count these cubes, if you could count one cube per second for 24 hours per day until they were all counted?

____

Imagine the same one meter box. Imagine filling it with one-dollar bills. How many one dollar bills could fit inside this box? Would it be more or less than one million?
(Hint: Use a ream of paper to help think about the height of a dollar bill.) __________________

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Additional Activities: The Volume of Paper Cylinders

The formula **V** = **l** x **w** x **h** is useful for finding the volume of a rectangular prism, but how do we find the volume of a cylinder? The answer is to take the area of the base times the height, where the area of the base is the area of the circle (πr^2 square units). The answer will be in cubic units.

Paper cylinders.
Take two pieces of 8.5"x11" paper. Tape one together into a cylinder along the 11" edges. Tape the other together along the 8.5" edges. Will the two cylinders have the same volume or different volumes? If they are different, which one has greater volume and by how much?

Packaging Problems

Packing and packaging problems are a good way to continue the discussion of capacity . What percentage of space is wasted when a round pizza is packaged in a square box? (Assume the pizza touches the edge of the box at four points.)

. ____

How is this related to the amount of wasted space in the rectangular carton that holds twelve soda cans?

Why are containers and packages the shapes that they are?

Rounded containers are better for micro-waving. Rectangular containers stack better. Which packages have the largest capacity for the amount of material used? _____________________
Explain your answer.

____

____

____

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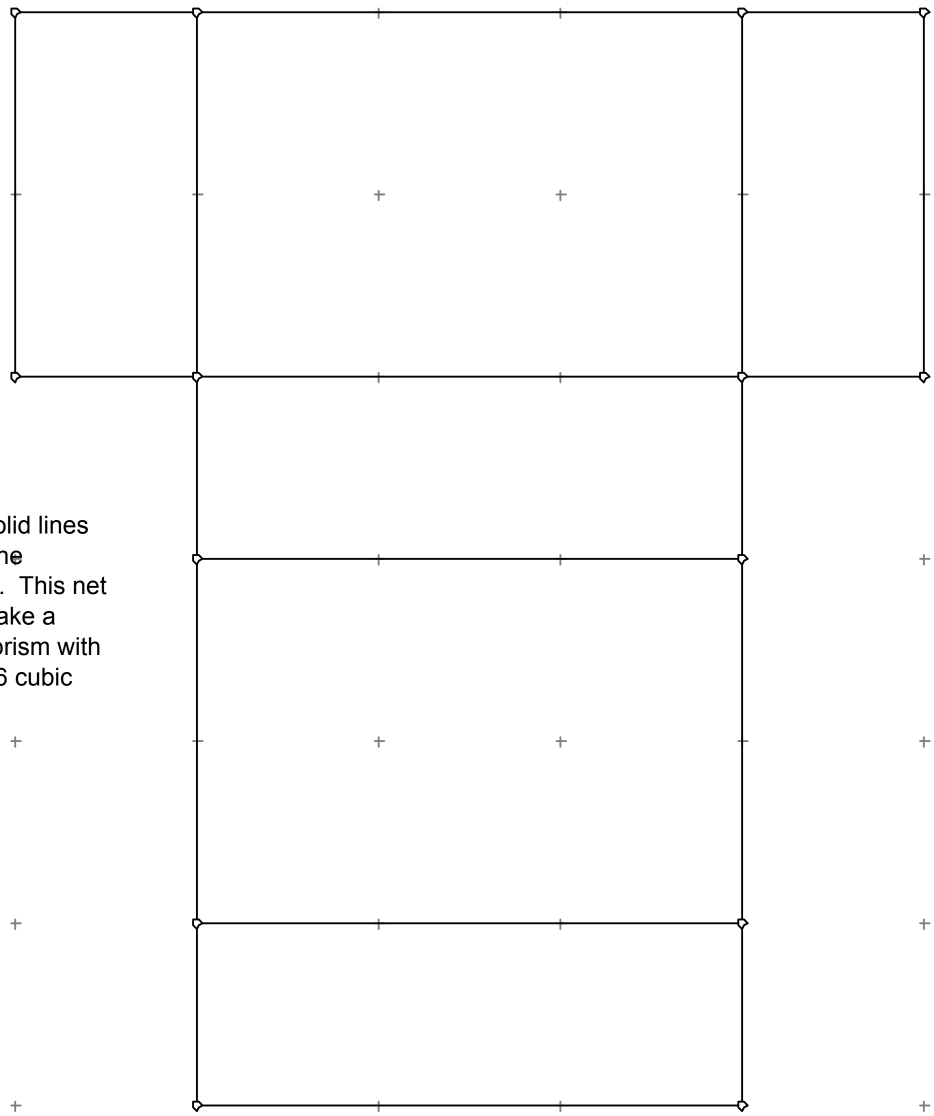
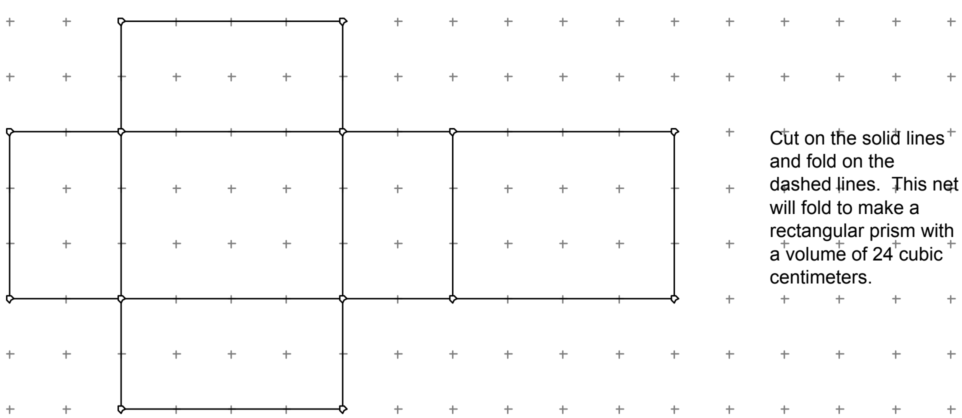
Rectangular Prisms with Six Closed Faces



Cut on the solid lines and fold on the dashed lines. This net will fold to make a rectangular prism with a volume of 6 cubic inches.



Cut on the solid lines and fold on the dashed lines. This net will fold to make a rectangular prism with a volume of 24 cubic centimeters.

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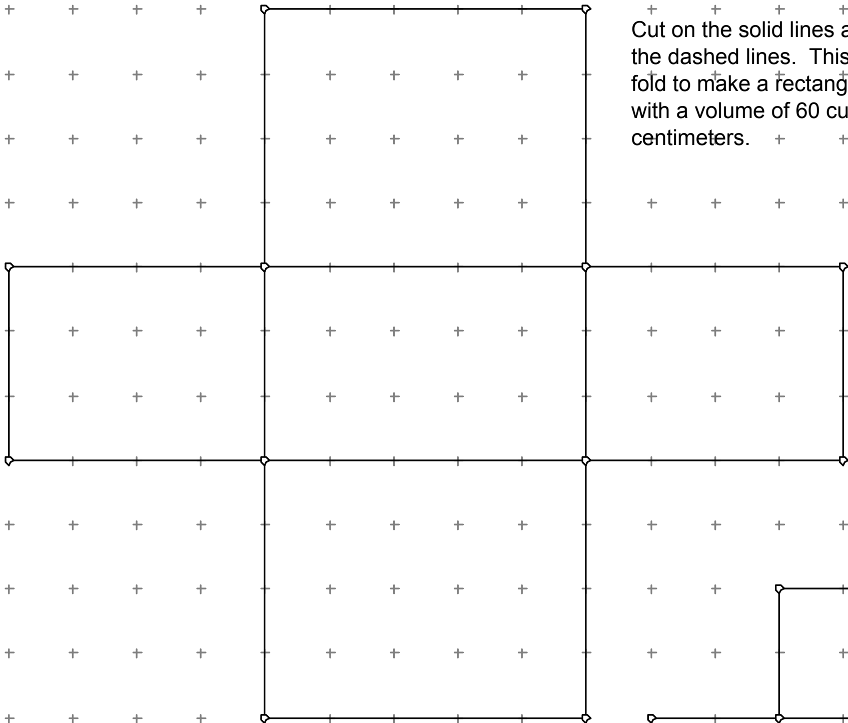
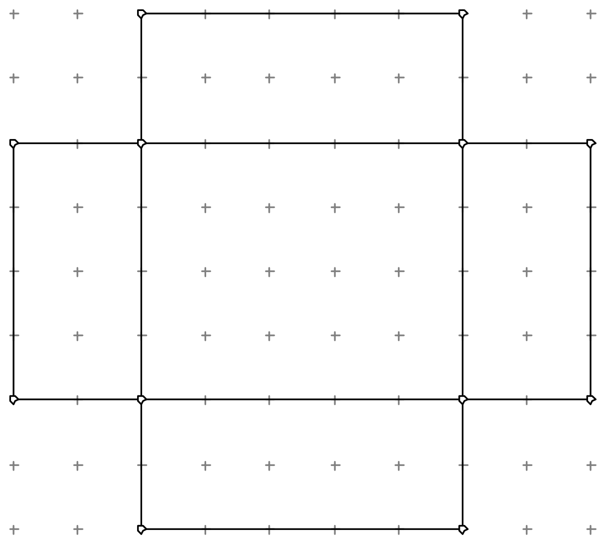
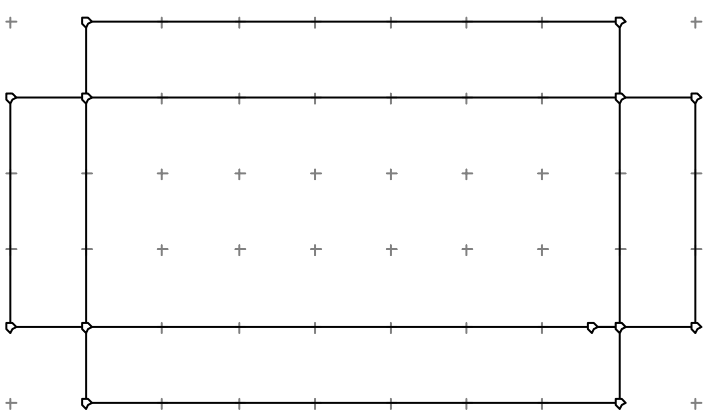
Rectangular Prisms with the Tops Open



Cut on the solid lines and fold on the dashed lines. This net will fold to make a rectangular prism with a volume of 60 cubic centimeters.

Cut on the solid lines and fold on the dashed lines. This net will fold to make a rectangular prism with a volume of 40 cubic centimeters.





Cut on the solid lines and fold on the dashed lines. This net will fold to make a rectangular prism with a volume of 21 cubic centimeters.

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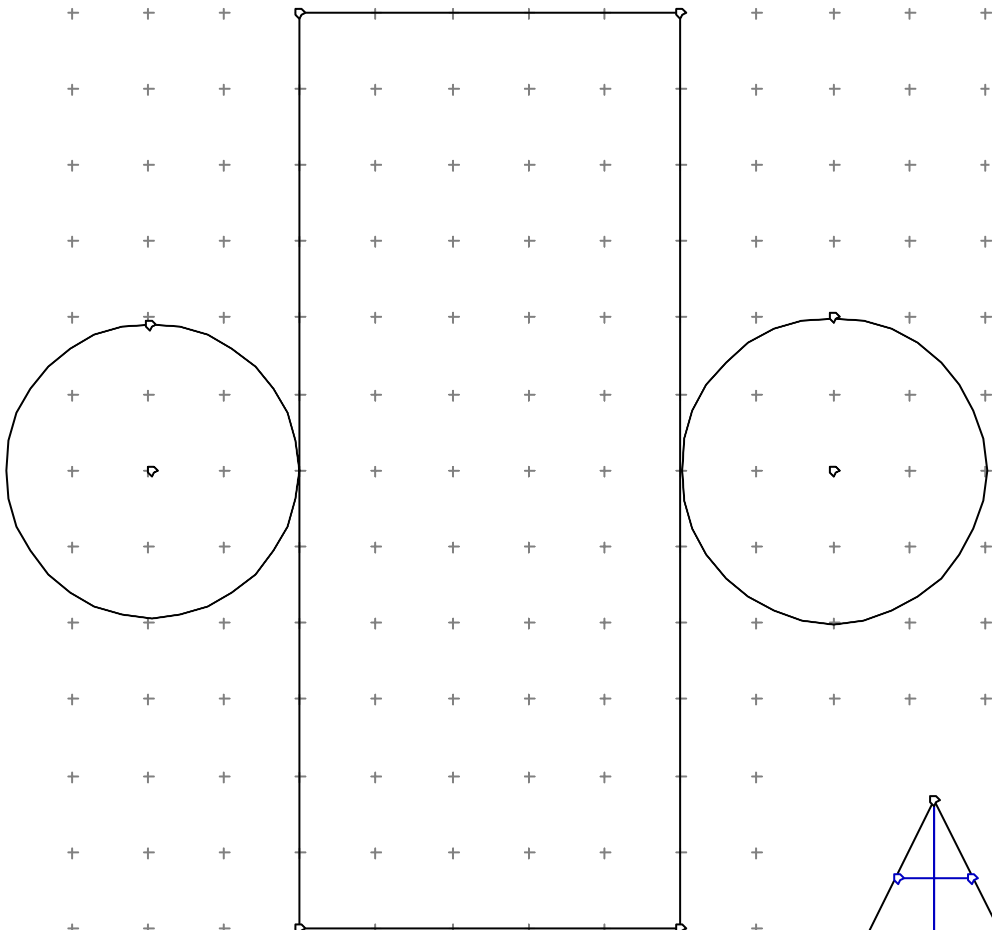
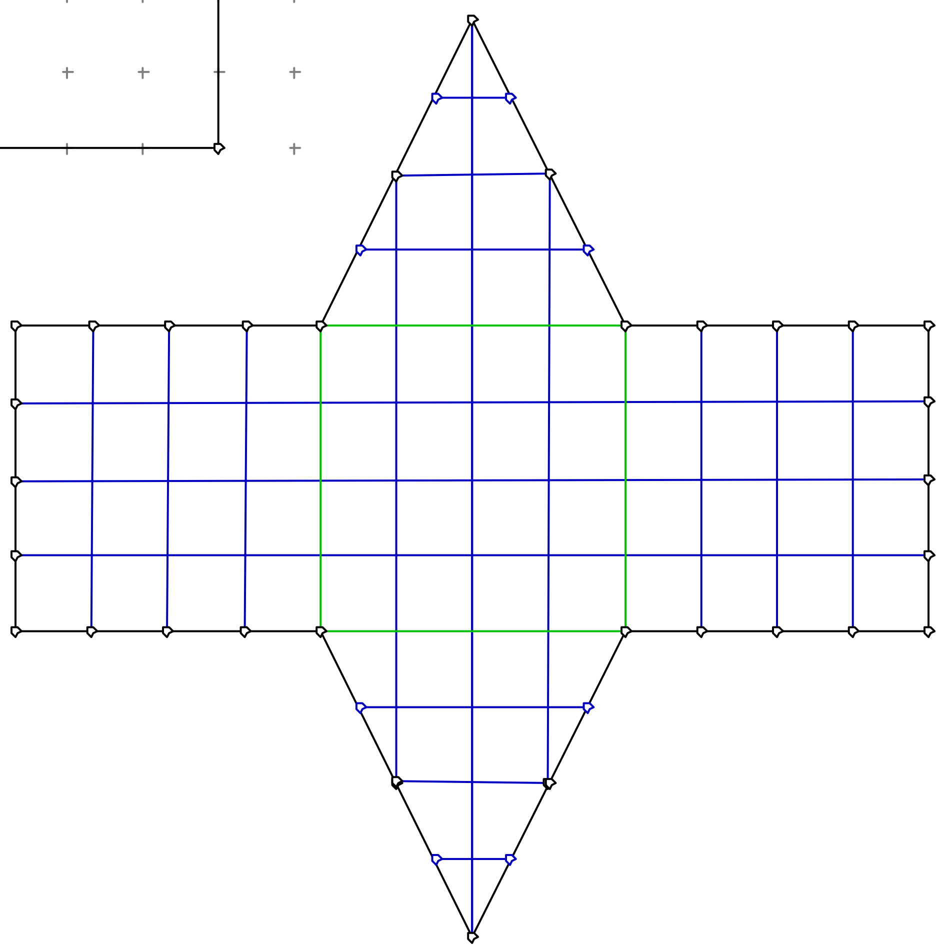
Nets for Other Prisms and Cylinders



This net will fold to make a cylinder with a volume of approximately 62.8 cubic centimeters.



Cut on the black lines and fold on the green lines. This net will fold to make a triangular prism with a volume of 32 cubic centimeters.

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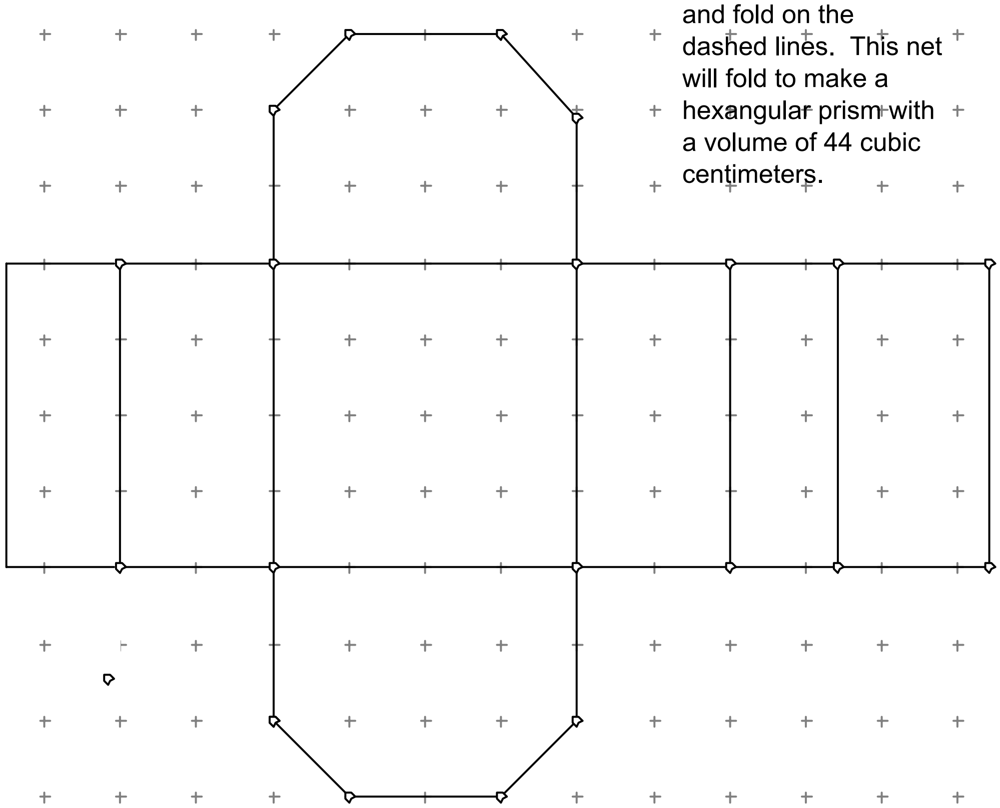
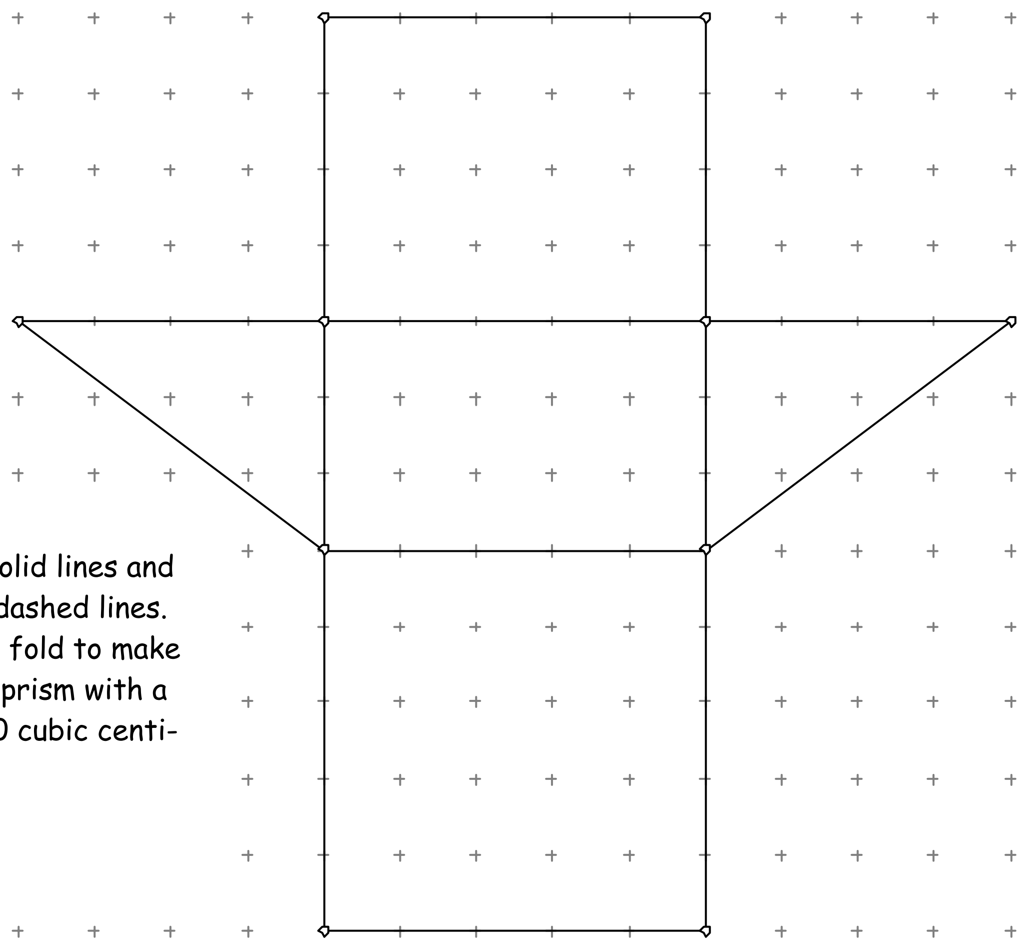
Appendix F (continued)

Cut on the solid lines and fold on the dashed lines. This net will fold to make a hexangular prism with a volume of 44 cubic centimeters.





Cut on the solid lines and fold on the dashed lines. This net will fold to make a triangular prism with a volume of 30 cubic centi-meters.

Email questions and comments to
m2t2@mail.mste.uiuc.edu

Measurement