M2T2

Expect The Unexpected:
Probability, Data, and Statistics

Statistics

State Goal 10: Collect, organize, and analyze data using statistical methods: predict results: interpret uncertainty using concepts in probability.

Statement of Purpose

We live in a data-filled world. Newspapers and magazines are full of examples of how politicians, educators, environmentalists, or others use data such as statistics and probability. Opinion polls, stock prices, tax rates, and weather predictions are increasingly part of our daily lives. A basic understanding of probability and statistics makes it possible to understand everything from batting averages to the weather report.

The activities in this module will engage the students in a variety of tasks that will cause them to collect data, predict results, compare their experimental results to theoretical findings, and find appropriate ways to display data.

Overview: This module deals first with some ideas about probability. It then deals with statistics. As will be shown, both topics are linked together in many ways.

Connections to the Illinois Learning Standards:

Standard 10.A - Organize, describe, and make predictions from existing data Throughout the module, participants are asked to organize their data and make predictions based on the results of data gained through experiments or data they are given.

Standard 10.B - Formulate questions, design data collection methods, gather and analyze data and communicate findings Participants in this module are asked to collect data and use appropriate graphs to interpret and communicate their results.

Standard 10.C – Determine, describe, and apply the probability of events This module begins with several activities involving probability: What's In the Bag, Is Rachel's Game Fair, and Luck of the Throw are all explorations of probability.

Table of Contents

M2T2

Materials

MATERIALS LIST

Minimal:

- pencils
- paper bag for each group,
- 20 predetermined colored centimeter cubes for each bag (some yellow, green, red, blue)
- 2 dice for each group
- 2 Red marbles for each group
- 3 Blue marbles for each group
- adding machine tape
- Yarn
- unifix cubes (classroom set)
- Index cards with the following terms printed: *Median, Minimum, Maximum, 3rd Quartile, 1st Quartile*
- enough pennies and nickels for each student in the class
- Sticky Dots
- Several Measuring tapes
- Markers
- Chart Paper for Wall Scatter Plot Activity

Optimal List will also include:

- An Internet Browser

Technology Extensions

For the Teacher:

- **The Math Forum… Probability and Statistics**
  http://mathforum.org/probstat/probstat.html
- **Intro to Probability**
  http://www-math.bgsu.edu/~albert/m115/probability/outline.html
- **The Math Forum… Collecting Data**
  http://mathforum.org/workshops/usi/dataproject/
- **MSTE University of Illinois**
  http://www.mste.uiuc.edu/

Technology Extensions:

Note: Most of these site require Macromedia Shockwave Player.
It is a free plugin: http://sdc.shockwave.com/shockwave/download/frameset.fhtml?

For the Student:

***Online Graphing/Data Collection Links*:**

- **Gene's Bike Shop** (information about making and using graphs)
  http://www.bbc.co.uk/education/revisewise/maths/data/11_act.shtml
- **Tables and Graphs (tutorial about graphs)**
  http://pittsford.monroe.edu/jefferson/calfieri/graphs/TabGraphMain.html
- **Create a Graph** (make a graph online)
  http://nces.ed.gov/nceskids/graphing/index.asp
- **Histograms (tutorial)**
  http://www.bbc.co.uk/schools/gcsebitesize/maths/datahandlingh/histogramsrev1.shtml
- **Data Picking** (frequency tables, charts, and graphs)
  http://www.bbc.co.uk/education/mathsfile/shockwave/games/datapick.html
- **Train Race** (mean, median, and range)
  http://www.bbc.co.uk/education/mathsfile/shockwave/games/train.html
- **Data Handling Intermediate** (tutorial)
  http://www.bbc.co.uk/schools/gcsebitesize/maths/datahandlingfi/index.shtml

***Online Probability Links*:**

- **Fish Tank**
  http://www.bbc.co.uk/education/mathsfile/shockwave/games/fish.html
- **Mrs. Glosser's Math Goodies…Probability**
  http://www.mathgoodies.com/lessons/toc_vol6.shtm
- **What Are Your Chances?**
  http://nces.ed.gov/NCESKids/Probability/
- **Tree Diagram** (tutorial)
  http://www.bbc.co.uk/schools/gcsebitesize/maths/datahandlingih/probabilityirev2.shtml

M2T2

Instructor Page

What's in the Bag?

Is it possible for students to predict the contents of a bag without knowing what is in the bag?

A **Sample** is a group of data that represents the entire group's population.

Experimental probability is based on the results of a series of trials

Theoretical probability is based on the number of favorable outcomes divided by the number of possible outcomes in the sample set.

By participating in this activity, students are given the opportunity to predict outcomes by performing random samples with replacement. They will *collect, record, and analyze* the data in order to make a prediction about what colored cubes are in the bag.

They will also use proportional reasoning skills.
For example: Suppose the group chooses to make 60 samples and the results of the sampling are as follows:

Red cubes	18
Blue cubes	24
Yellow cubes	6
Green cubes	12

The students would hopefully determine that since they only have 20 cubes in their bag that the sample of 60 would be three times as many as they have. They might therefore conclude that there are 6 red, 8 blue, 2 yellow and 4 green in their bag. The actual data probably will not be this neat and so they will have to make decisions in their predicting process. In the final analysis, it is hoped that the students will gain understanding of the importance of sample size and how to make good predictions based on the random sample.

Materials: paper bag per group, 20 predetermined colored centimeter cubes for each bag (some yellow, green, red, blue)

Guidepost:

- Introduce the ideas of making a prediction by sampling with replacements. Determine as a class the number of replacements necessary to predict how many of each color are in the bag *(Multiple of 20.. Perhaps 60)*
- Each group will be given a bag with 8 green cubes, 8 red cubes, 3 yellow cubes, and 1 blue cube. *(Do not tell them how many of each color is in the bag)*
- Instruct each group to:
  - Pick one cube from the bag, tally it by color, and then replace it back in the bag.

Yellow Cube	Blue Cube	Green Cube	Red Cube

M2T2

Participant Page

As you work through this activity, you should become familiar with the following terms:

- Sample
- Experimental Probability
- Theoretical Probability

What's in the Bag?

Your group will be given a bag containing 20 colored cubes (some red, green, yellow, and blue).

- WITHOUT looking, you are to:
  1. **Take out one cube**
  2. **Tally its color on the chart**
  3. **Return the cube to the bag**
  4. **Repeat steps 1-3 _____ times (to be determined by class discussion)**

Record your data on the following tally chart:

Yellow Cube	Blue Cube	Green Cube	Red Cube

1. **Predict how many of each color is in your bag. Remember that there are 20 colored cubes in the bag. There is at least one cube of each color.**

   - How many Yellow cubes? _____
   - How many Blue cubes? _____
   - How many Green cubes? _____
   - How many Red cubes? _____

2. **How did you determine your results from your sampling?**

_____

_____

_____

_____

Instructor Page

What's in the Bag? (continued)

Discussion Questions: ***Before students actually look in bag***

1. Ask: Why would using a multiple of 20 for the number of tries be helpful? (Could use proportions to determine answer)

- **Have each group predict how many of each colored cube is in their bag (___ out of 20 are yellow ...are green ...are red ...are blue)**
- **Discuss group results**

2. Collect actual tallied data from each group to determine the class results (Divide tally total for each color by total tallies of all 4 colors combined)

- **Each group will be given a chance to revise their prediction based upon whole class data**
- **As a class, determine how many cubes are in the bag using the class data**

3. Have each group take the blocks out of the bag to see how many of each there were in the bag

4. Compare class's *experimental results* with *theoretical results*

- To get class's *experimental results*: Divide class's tally total for each color by total tallies of all 4 colors combined
- Compare to *theoretical results*
  - 8/20 or 2/5 should be green (.4)
  - 8/20 or 2/5 should be red (.4)
  - 3/20 should be yellow (.15)
  - 1/20 should be blue (.05)

5. How could the class improve the accuracy of the experimental results? (Carry out more trials)

M2T2

Participant Page

What's in the Bag? (continued)

3. **What was the class prediction?**
   - How many Yellow cubes? _____
   - How many Blue cubes? _____
   - How many Green cubes? _____
   - How many Red cubes? _____

4. **What was the class's experimental probability for each color?**
   - Yellow _____
   - Blue _____
   - Green _____
   - Red _____

5. **How many of each color were actually in the bag?**
   - How many Yellow cubes? _____
   - How many Blue cubes? _____
   - How many Green cubes? _____
   - How many Red cubes? _____

6. **What was the theoretical probability for each color?**
   - Yellow _____
   - Blue _____
   - Green _____
   - Red _____

7. **What could be done to make the class's experimental prediction, more closely match the theoretical prediction?**

_____

_____

M2T2

Instructor Page

Luck of the Throw Game

Experimental probability is based on the results of a series of trials

Theoretical probability is based on the number of favorable outcomes divided by the number of possible outcomes.

A **Fair Game** occurs when all players have an equal chance of winning.

A **Tree Diagram** is a network of points connected by line segments.

Alonzo and Lakeisha designed a gamed called Luck of the Throw. To play, you roll two dice, one at a time.

- **If the first die lands on an *even number*, you lose.**
- **If it lands on an *odd number*, you get to roll the second die.**
- **If the second die lands on an *odd number*, you win.**
- **If it lands on an *even number*, you lose.**

What is the probability of winning this game? Is this a fair game?

Material: 2 die per group

Guideposts:

Introduce Alonzo and Lakeisha's dice game.

- Discuss the idea of fair games. If a game is fair, it has an equal chance of winning or losing.
- How can we determine if this game is fair? (Play game)
- Have the students actually play the game ten times, tally results, Combine the class data to get a larger sample set

Mark each box as a W for win or an L for loss.
To win, both throws must be odd. If either throw is an even number, player loses.

	Trial	Trial	Trial	Trail	Trail	Trial	Trial	Trial	Trial	Trail
Player										
Player										

M2T2

Participant Page

Become familiar with the following terms:

- Tree Diagram
- Fair Game
- Experimental probability
- Theoretical probability

Luck of the Throw Game

Alonzo and Lakeisha designed a gamed called Luck of the Throw. To play, you roll two dice, one at a time.

- If the first die lands on an *even number*, you lose.
- If it lands on an *odd number*, you get to roll the second die.
- If the second die lands on an *odd number*, you win.
- If it lands on an *even number*, you lose.

Is this a fair game? Why or why not?

Remember to win:

You must throw an odd number on both rolls.

A Solution:

We can actually play the game to see if it is fair. If fair, there should be an almost equal amount of wins and losses.

Mark each box as a W for win or an L for loss.
To win, both throws must be odd. If either throw is an even number, player loses.

	Trial	Trial	Trial	Trail	Trail	Trial	Trial	Trial	Trial	Trail
Player										
Player										

Combine the class data to determine the % of wins and % of losses.

% of wins _____ (number of wins / total number of games played)

% of losses _____ (number of losses / total number of games played)

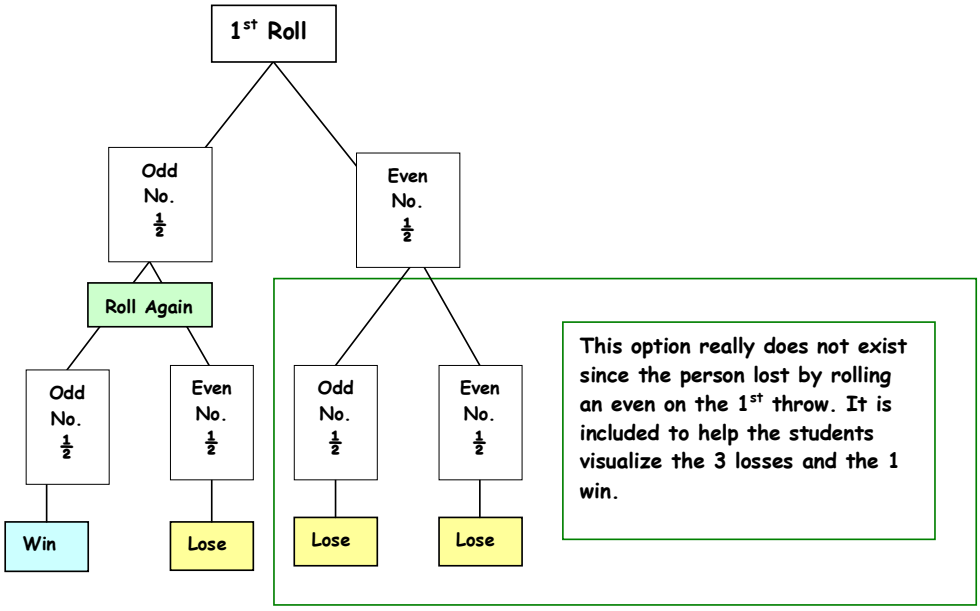
Instructor Page

Luck of the Throw (continued)

Class Discussion:

1. **How can we determine the probability of being able to win this game?**
   - Divide total wins by total number of games played (change to %)
   - Divide total losses by total games played (change to %)

2. **How can we determine the Theoretical probability using a tree diagram?**



This activity involves a two-tiered tree diagram.
All of the possible outcome for this games are:

Probability of *Winning* is 1/4.
Probability of *Losing* is 3/4

3. **Have students explain why game is not fair.** (Game is *not* fair since there is *three times* more of a chance to lose.)

M2T2

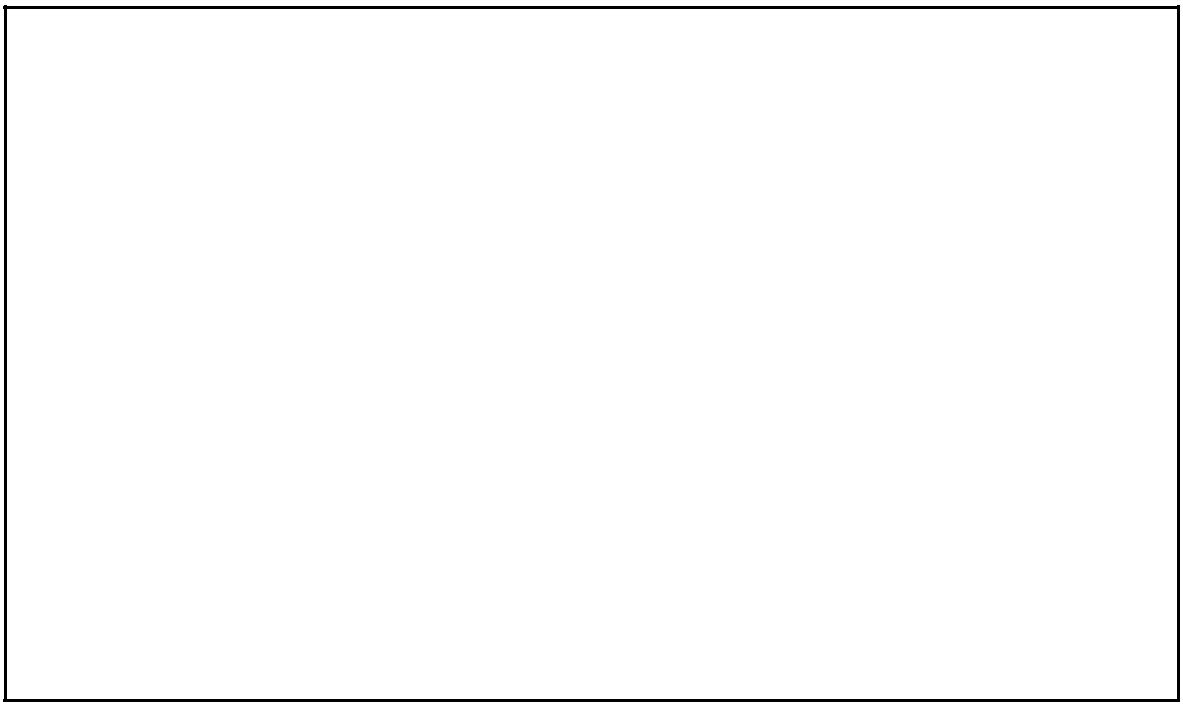
Participant Page

Luck of the Throw (continued)

1. After combining class results, do you think this game is fair? Why or why not? _____

_____

2. Let's determine the theoretical probability of this game using a tree diagram. Make the tree diagram by following the directions given by the instructor:



3. After looking at the theoretical probability, do you think this a fair game? Why or why not?

_____

_____

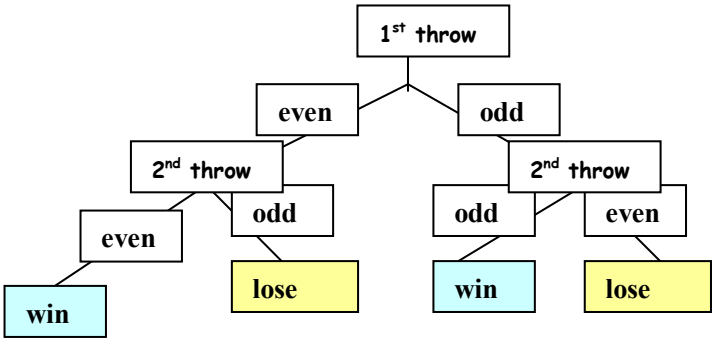
Instructor Page

Luck of the Throw (continued)

4. **If 52 people played this game, how many would you expect to win?** (1/4 of 52 is 13)

5. **Have students change the rules to make this a fair game.**
   - One example for a fair game:
     - Player throws the two dice, one at a time.
     - To win, both throws must be the same (both even number or both odd numbers)
     - To loose, both throws are different (one is even number and one is odd number)

6. **Have students make a tree diagram to show game is fair**



There is an equal chance to win or loose.

Online Resources: **Web Dice (to be used instead of actual dice)**
http://www.mste.uiuc.edu/activity/webdice.html

M2T2

Participant Page

Luck of the Throw (continued)

4. If 52 people played this game, how many would you expect to win? Explain.

_____

5. How can you change the rules to make it a fair game?

_____

_____

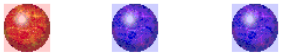
_____

6. Make a tree diagram and prove that your new game is a fair game.

M2T2

Instructor Page

Is Rachel's Game Fair?
Game 1



Review the following terms:

- Fair Game
- Experimental Probability
- Theoretical Probability

Rachel asked Holly to play this game.
She has 1 red and 2 blue marbles in a bag. She tells Holly to pull two marbles out of the bag without looking. If the marbles are the same color, Holly will win. If the marbles are different colors then Rachel will win. Is this a fair game? Why or why not?

To win:

- **Rachel must draw two *different* colored marbles.**
- **Holly must draw two marbles the *same* color.**

The main idea here is to determine if a game is fair. *If a game is fair, there must be an equal chance of winning or losing.*

The students will begin by carrying out experimental probability and then check results compared to the theoretical probability

Materials: paper bag, 2 Red marbles, 3 Blue marbles for each group,

Guideposts:

1. Introduce Rachel's problem to the class.
   - **Discuss how the problem can be modeled. *(by simulating the activity with marbles)***
   - **How can we determine if Game 1 is fair?**
     - One way would be for pairs of students to actually play the game 50 times.
     - Each student colors in the circles the same color as the marble drawn. Each will pick two marbles out of the bag.
     - The box will be marked with a **W** if the game is won and a **L** if game is lost.

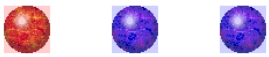
M2T2

Participant Page

As you work through this activity you should become familiar with the following terms:

- **Experimental probability**
- **Theoretical probability**
- **Fair Game**

Is Rachel's Game Fair?
Game 1



Rachel asked Holly to play this game. She has 1 red and 2 blue marbles in a bag. She tells Holly to pull two marbles out of the bag without looking. If the marbles are the same color, Holly will win. If the marbles are different colors then Rachel will win. Is this a fair game? Why or why not?

To win:

- **Rachel must draw two *different* colored marbles.**
- **Holly must draw the *same* colored marbles .**

A Solution:

How could we solve this problem? *One way would be to do an experiment where we actually play Game 1 with 1 Red marble and 2 Blue marbles.* ***Player 1 wins if both marbles are the same color. Player 2 wins if both colors are different.***

- **Determine who will be Player 1 and Player 2. For EACH trial:**
  - ***Player 1*** **picks two marbles and records the colors on the chart. A W for win (both colors the same) and an L for loss (both colors different) is marked on the table.**
  - ***Player 2*** **picks two marbles and records the colors on the chart. A W for win (both colors different) and an L for loss (both colors the same) is marked on the table.**

Instructor Page

Game 1
Talley Summary Sheet

	Trial 1	Trial 2	Trial 3	Trail 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial
Player 1										
Player 2										

	Trial 11	Trial 12	Trial 13	Trial 14	Trial 15	Trial 16	Trial 17	Trial 18	Trial 19	Trial
Player 1										
Player 2										

	Trial 21	Trial 22	Trial 23	Trial 24	Trial 25	Trial 26	Trial 27	Trial 28	Trial 29	Trial
Player 1										
Player 2										

	Trial 31	Trial 32	Trial 33	Trial 34	Trial 35	Trial 36	Trial 37	Trial 38	Trial 39	Trial
Player 1										
Player 2										

	Trial 41	Trial 42	Trial 43	Trial 44	Trial 45	Trial 46	Trial 47	Trial 48	Trial 49	Trial
Player 1										
Player 2										

2. Collect data from all of the groups and discuss the results. (If the game is fair, both player 1 and player 2 should have about the same number of wins) *Class should determine that the game is NOT fair.*

M2T2 Participant Page

Game 1
Talley Summary Sheet

	Trial 1	Trial 2	Trial 3	Trail 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial
Player										
Player										
	Trial	Trial	Trial	Trial	Trial	Trial	Trial	Trial	Trial	Trial
Player										
Player										
	Trial	Trial	Trial	Trial	Trial	Trial	Trial	Trial	Trial	Trial
Player										
Player										
	Trial	Trial	Trial	Trial	Trial	Trial	Trial	Trial	Trial	Trial
Player										
Player										
	Trial	Trial	Trial	Trial	Trial	Trial	Trial	Trial	Trial	Trial
Player										
Player										

Does Game 1 seem to be a fair game? Why or why not?

____

____

Instructor Page

Is Rachel's Game Fair?
Game 2

3. **GAME 2… Would adding another Red marble to the bag, make this a fair game?** ***(There are now 2 Red and 2 Blue Marbles in the bag) (Many will think that is fair because there is an equal number of Red and Blue marbles)***

- **Have students actually play this new game 50 times.**
- **Color in marble color and mark each box as a win (W) or a loss (L)**

	Trial 1	Trial 2	Trial 3	Trail 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10
Player 1										
Player 2										
	Trial 11	Trial 12	Trial 13	Trial 14	Trial 15	Trial 16	Trial 17	Trial 18	Trial 19	Trial 20
Player 1										
Player 2										
	Trial 21	Trial 22	Trial 23	Trial 24	Trial 25	Trial 26	Trial 27	Trial 28	Trial 29	Trial 30
Player 1										
Player 2										
	Trial 31	Trial 32	Trial 33	Trial 34	Trial 35	Trial 36	Trial 37	Trial 38	Trial 39	Trial 40
Player 1										
Player 2										
	Trial 41	Trial 42	Trial 43	Trial 44	Trial 45	Trial 46	Trial 47	Trial 48	Trial 49	Trial 50
Player 1										
Player 2										

4. **Collect the data from all of the groups and discuss class results.**
(If the game is fair, both players should have an equal number of wins...Game is not fair)

M2T2

Participant Page

Is Rachel's Game Fair?
Game 2

Game 2) The bag now has 2 Blue marbles and 2 Red marbles. Will this be a fair game? Why or why not?

_____

Actually play this game 50 times using same rules as Game 1
Mark each player's box as a W for win or an L for Loss

	Trial 1	Trial 2	Trial 3	Trail 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10
Player 1										
Player 2										

	Trial 11	Trial 12	Trial 13	Trial 14	Trial 15	Trial 16	Trial 17	Trial 18	Trial 19	Trial 20
Player 1										
Player 2										

	Trial 21	Trial 22	Trial 23	Trial 24	Trial 25	Trial 26	Trial 27	Trial 28	Trial 29	Trial 30
Player 1										
Player 2										

	Trial 31	Trial 32	Trial 33	Trial 34	Trial 35	Trial 36	Trial 37	Trial 38	Trial 39	Trial 40
Player 1										
Player 2										

	Trial 41	Trial 42	Trial 43	Trial 44	Trial 45	Trial 46	Trial 47	Trial 48	Trial 49	Trial 50
Player 1										
Player 2										

Was Game 2 fair? How do you know?

_____

_____

Instructor Page

Is Rachel's Game Fair?
Game 3

5. **Game 3… What would happen if the bag held 3 Blue and 1 Red marble? Would that be a fair game?** ***(Many will think this is not a fair game since there are 3 times more Blue marbles than Red marbles)***

- **Have students actually play this game 50 times.**
- **Color in marble color and mark each box as a W for win or an L for loss**

	Trial 1	Trial 2	Trial 3	Trail 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10
Player 1										
Player 2										

	Trial 11	Trial 12	Trial 13	Trial 14	Trial 15	Trial 16	Trial 17	Trial 18	Trial 19	Trial 20
Player 1										
Player 2										

	Trial 21	Trial 22	Trial 23	Trial 24	Trial 25	Trial 26	Trial 27	Trial 28	Trial 29	Trial 30
Player 1										
Player 2										

	Trial 31	Trial 32	Trial 33	Trial 34	Trial 35	Trial 36	Trial 37	Trial 38	Trial 39	Trial 40
Player 1										
Player 2										

	Trial 41	Trial 42	Trial 43	Trial 44	Trial 45	Trial 46	Trial 47	Trial 48	Trial 49	Trial 50
Player 1										
Player 2										

6. **Collect the data from all of the groups and discuss class results.**
(If the game is fair, both players should have an equal number of wins...Game is not fair)

M2T2

Participant Page

Is Rachel's Game Fair?
Game 3

Game 3) The bag now has 3 Blue marbles and 1 Red marble. Do you think that this is going to be a fair game? Why or why not?

Actually play this game 50 times using same rules as Game 1
Mark each player's box as a W for win or an L for Loss

	Trial 1	Trial 2	Trial 3	Trail 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10
Player 1										
Player 2										

	Trial 11	Trial 12	Trial 13	Trial 14	Trial 15	Trial 16	Trial 17	Trial 18	Trial 19	Trial 20
Player 1										
Player 2										

	Trial 21	Trial 22	Trial 23	Trial 24	Trial 25	Trial 26	Trial 27	Trial 28	Trial 29	Trial 30
Player 1										
Player 2										

	Trial 31	Trial 32	Trial 33	Trial 34	Trial 35	Trial 36	Trial 37	Trial 38	Trial 39	Trial 40
Player 1										
Player 2										

	Trial 41	Trial 42	Trial 43	Trial 44	Trial 45	Trial 46	Trial 47	Trial 48	Trial 49	Trial 50
Player 1										
Player 2										

Instructor Page

Is Rachel's Game Fair? (continued)

Discussion of activity:

Let's analyze the three games played using *theoretical probability.*

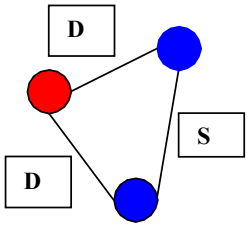
Both the networked combination display and the tree diagram will be shown to the students.

Game 1:

Students will determine the theoretical probability of Game 1:

- Students will color the marbles.
- Students will connect marbles with a line to determine the possibilities of drawing the same or different colored marbles.

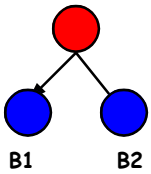
Networked combination display:

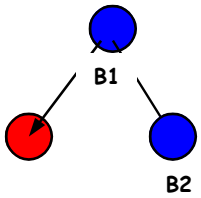


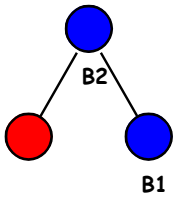
D = different
S= Same

Is this game fair?
No…
Your chance of getting two different colors is 2 out of 3.
Your chance of getting both the same color is 1 out of 3

Tree diagram display:







*note B1 and B2 represent the 2 different blue marbles

Is this game fair after looking at tree diagram? No…

2/6 ways to get same colored marbles
4/6 ways to get different colored marbles.

M2T2

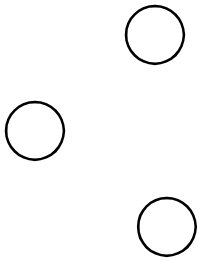
Participant Page

Is Rachel's Game Fair? (continued)

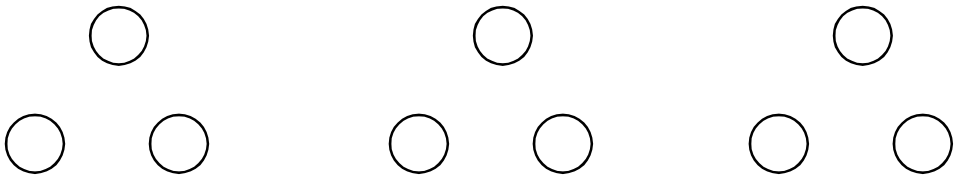
Let's determine the theoretical probability for each of the three games. Follow the directions given by the instructor:

Game 1

Networked combination display:



Tree diagram display:



Is Game 1 fair? How do you know?

_____

_____

M2T2

Instructor Page

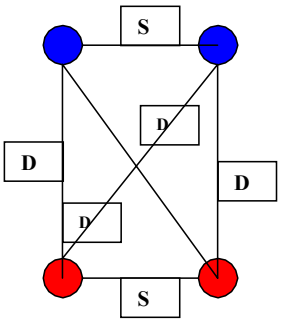
Is Rachel's Game Fair? (continued)

Game 2:

Students will determine the theoretical probability of Game 2.

- Students will color the marbles.
- Students will connect marbles with a line to determine the possibilities of drawing the same or different colored marbles.

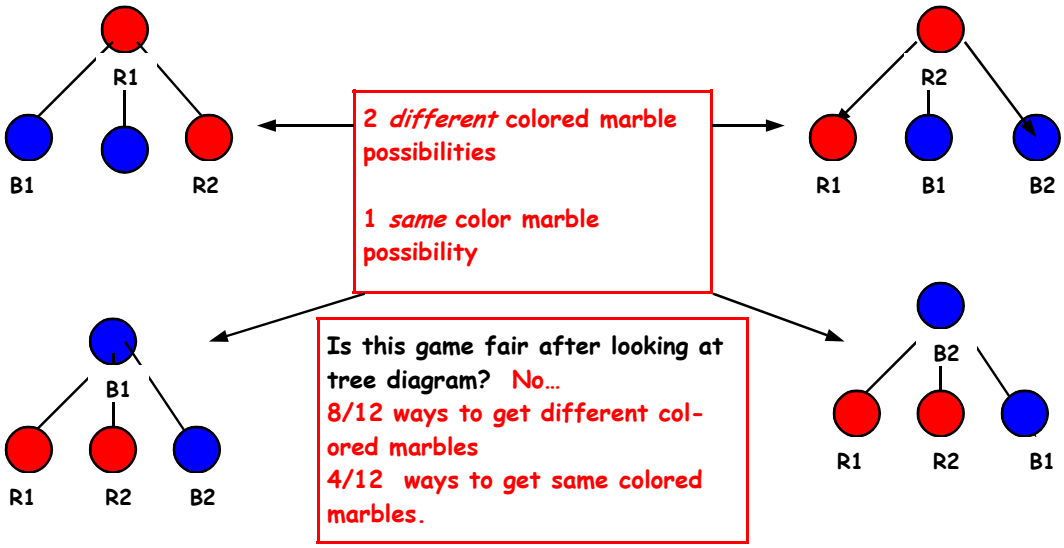
Networked combination display:



D = different
S= Same

Is this game fair?
No…
Your chance of getting two different colors is 4 out of 6.
Your chance of getting both the same color is 2 out of 6.

Tree Diagram Display:



Is this game fair after looking at tree diagram? No…
8/12 ways to get different colored marbles
4/12 ways to get same colored marbles.

M2T2

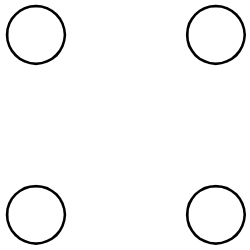
Participant Page

Let's determine the theoretical probability for Game 2

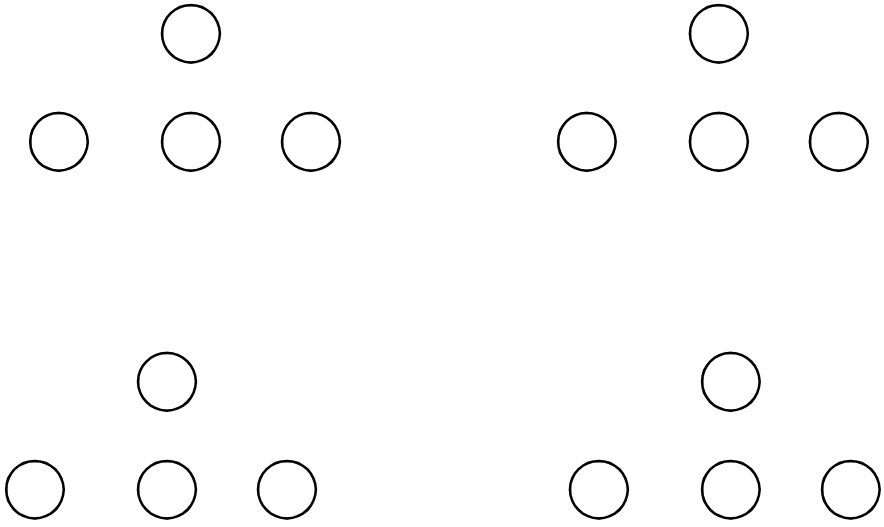
Follow the directions given by the instructor:

Game 2

Networked combination display:



Tree diagram display:



Is Game 2 fair? How do you know?

_____

_____

Instructor Page

Is Rachel's Game Fair? (continued)

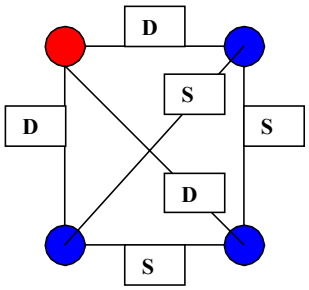
Discussion of activity (continued):

Game 3:

Students will determine the theoretical probability of Game 3.

- **Students will draw and color the 4 marbles.**
- **Students will connect marbles with a line to determine the possibilities of drawing the same or different colored marbles.**

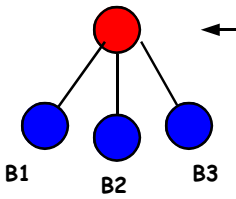
Networked combination display:



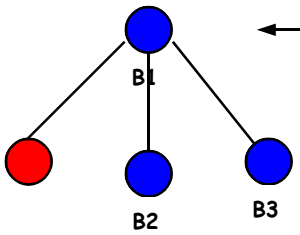
D = different
S= Same

Is this game fair?
Yes…
Your chance of getting two different colors is 3 out of 6.
Your chance of getting both the same color is 3 out of 6.

Tree Diagram Display:

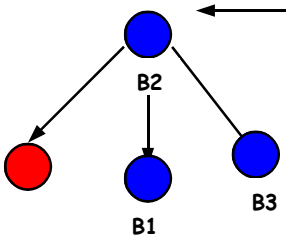


3 ***different*** color possibilities



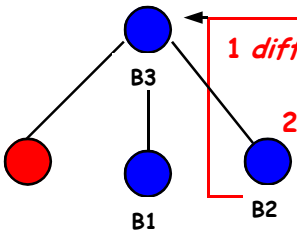
1 ***different*** color possibility

2 ***same*** color possibilities



1 ***different*** color possibility

2 ***same*** color possibilities



1 ***different*** color possibility

2 ***same*** color possibilities

Is this game fair after looking at tree diagram? Yes…
6/12 ways to get different colored marbles
6/12 ways to get same colored marbles.

M2T2

Participant Page

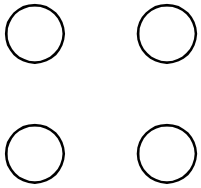
Is Rachel's Game Fair? (continued)

Let's determine the theoretical probability for Game 3

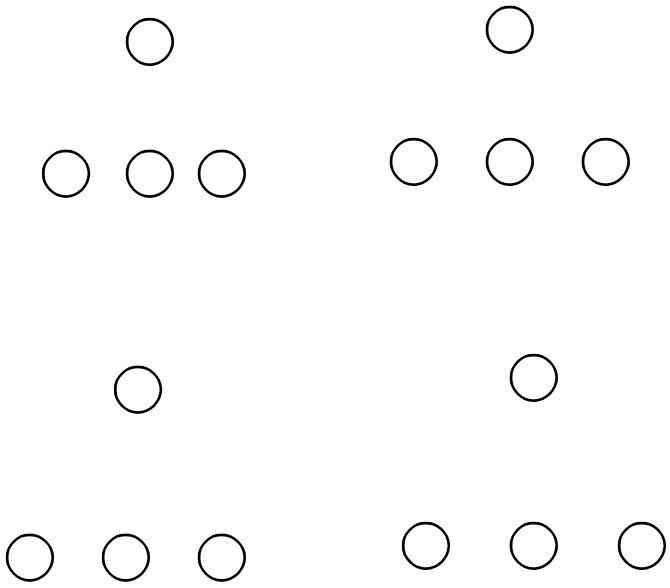
Follow the directions given by the instructor:

Game 3

Networked combination display:



Tree diagram display:



Is Game 3 fair? How do you know?

_____

_____

Instructor Page

Is Rachel's Game Fair? (continued)

Note to teacher:

By playing the game and tallying the results, the students are finding the *experimental probability*.

By analyzing each game, the students are using the *theoretical probability*.

Extension:

1. **What can be done to make the experimental probability more closely match the theoretical probability?** ***(Increase the number of trials for each game)***

2. **Looking for a pattern:**
   - **The 1st combination needed for a fair game will be 1 Red and 3 Blue marbles**
   - **The 2nd combination needed for a fair game will be 3 Red and 6 Blue marbles.**
   - **The 3rd combination needed for a fair game will be 6 Red and 10 Blue marbles?**
   - **How many Red marbles will be needed for the 5th fair game?**

Nth term	Number of Red
1	1
2	3
3	6
4	?
5	?
6	?

(1,3,6,?,?) [4th term will be 10, 5th term will be 15, 6th term will be 21]

- **Note the number of marbles to make a fair game will always be two consecutive numbers from this pattern (1,3…3,6…6,10…)**

3. **Have class discussion about triangular numbers (1,3,6,10,…,…,…)**

THIS PAGE INTENTIONALLY BLANK

M2T2

Instructor Page

- Mean is the mathematical average found by adding all the elements of the data set and then dividing by the number of elements in the set.
- Median is the middle number in a set of data, which is arranged in order.
- Mode is the most frequent or common element in the set of data.
- Range is the difference between the greatest and least values.

Stem and Leaf Plots

A table that shows groups of data arranged by place value

***Example*:**

Numbers in data set:

Coin Activity:


When organizing a set of data, such as the dates on pennies and nickels, it is helpful to first arrange the data in order. The stem-and-leaf table displays data in order from the least to greatest in an efficient manner. To make a stem and leaf table, one would place all the tens digits under the "stem" column. Then one would place all the units digits under the "leaf" column. If a unit occurs more than once, it needs to be repeated as a separate leaf every time it occurs.

The stem-and-leaf table allows someone to find the three measures of central tendency: *mean, median, and mode*.

Materials: enough pennies and nickels for each student in the class

Guideposts:

1. Each student will bring a penny and a nickel to the classroom.
2. Students will record the date of each coin on the overhead or the chalkboard.
3. Using the entire class data, make a stem-and-leaf table for each coin. This table shows data organized by place value.
   - Title the Stem-and-leaf Table
   - Find the oldest date on the coins _____
   - Find the newest date on the coins _____
   - Write the dates in order from oldest to newest.
   - The stem will be the number of tens. [the first three digits of the date]
   - Place the stem values from least to greatest in the table.
   - The leaves will be the units digit.
   - Write a key that explains how to read the stems and leaves.

M2T2

Participant Page

Stem-and-Leaf Plots
Coin Activity:

You should become familiar with the following terms:

- **Mean**
- **Median**
- **Mode**
- **Range**.

1. Each student will bring a penny and a nickel to the classroom.
2. Students will record the date of each coin on the overhead or the chalkboard.
3. Using the entire class data, make a stem-and-leaf table for each coin. This table shows data organized by place value.
4. Process for making class Stem-and-leaf Table
   - Title the Stem-and-leaf Table
   - Find the oldest date on the coins _____
   - Find the newest date on the coins _____
   - Write the dates in order from oldest to newest.
   - The stem will be the number of tens. [The first three digits of the date]
   - Place the stem values from least to greatest in the table.
   - The leaves will be the *units* digit.
   - The stem will be the *tens* value
   - Write a key that explains how to read the stems and leaves.

Sample coins: 1962, 1968, 1974, 1974, 1977, 1982, 1987, 1989, 1990, 1990, 1990, 1993, 1998, 1999, 2000, 2000, 2001

Sample Stem-and-Leaf Table

Stem	Leaves
196	2 8
197	4 4 7
198	2 7 9
199	0 0 0 3 8 9
200	0 0 1

Instructor Page

Stem-and-Leaf Plots
Coin Activity: (continued)

Sample coins: 1962, 1968, 1974, 1974, 1977, 1982, 1987, 1989, 1990, 1990, 1990, 1993, 1998, 1999, 2000, 2000, 2001

Sample Stem-and-Leaf Table

Stem	Leaves
196	2 8
197	4 4 7
198	2 7 9
199	0 0 0 3 8 9
200	0 0 1

Stem-and-Leaf Table for Classroom Pennies

Stem	Leaves

Stem-and-Leaf Table for Classroom Nickels

Stem	Leaves

M2T2

Participant Page

Stem-and-Leaf Plots
Coin Activity: (continued)

Stem-and-Leaf Table for Classroom Pennies

Stem	Leaves

Stem-and-Leaf Table for Classroom Nickels

Stem	Leaves

Where are the "stem" numbers coming from on the pennies?

_____

_____

_____

What does the "leaf" number tell you about the nickel?

_____

_____

_____

If a new student entered the room, which measure of central tendency is most likely to tell you what date is on the nickel in his pocket? _____

Instructor Page

Stem-and-Leaf Plots
Coin Activity: (continued)

4. Ask the participants where the numbers in the "stem" are coming from and what the "leaves" mean. (Stem is the number of tens. Leaves are number of units.)

5. Use the stem and leaf table to calculate the *mean, median, mode, and range* for each table.

6. Ask: If a new student were to enter the room, which measure of central tendency is most likely to predict the date on the nickel in his pocket? (mode)

You can use a *double stem-and-leaf plot* to compare two sets of data.

Classroom Pennies		Classroom Nickels
Leaves	Stem	Leaves

M2T2

Participant Page

Stem-and-Leaf Plots
Coin Activity: (continued)

You can use a *double stem-and-leaf plot* to compare two sets of data.

Fill in the following table from your classroom data.

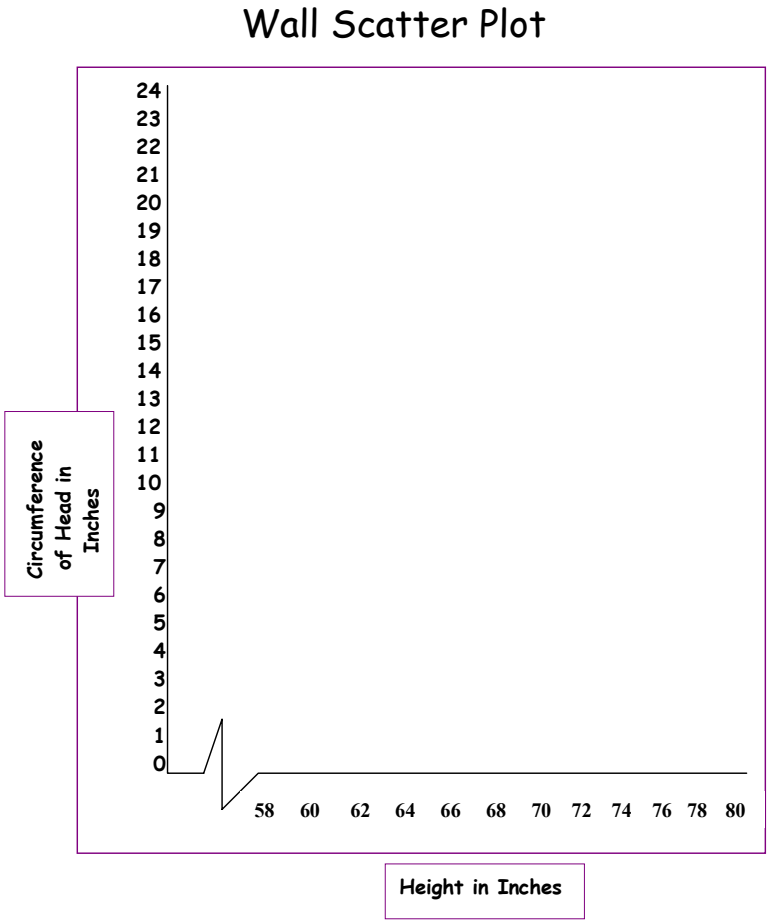
Classroom Pennies		Classroom Nickels
Leaves	Stem	Leaves

Scatter Plot Activity

Instructor Page

Scatter plot is a graph composed of data points. The coordinates represent the two characteristics that are measured for each piece of data.

Correlation is the strength of the relationships of the two variables being measured.



Materials: Several measuring tapes, *numbered* **Sticky Dots, Markers, Chart Paper for Wall Scatter Plot Activity**

Guidepost:

1. Have wall scatter plot chart displayed *before* participants begin activity.
2. As the participants enter the room, have them place *one* numbered sticky dot on wall chart that represents *both their height in inches* and *circumference of their head in inches.* (this activity will be discussed later in the lesson)

M2T2

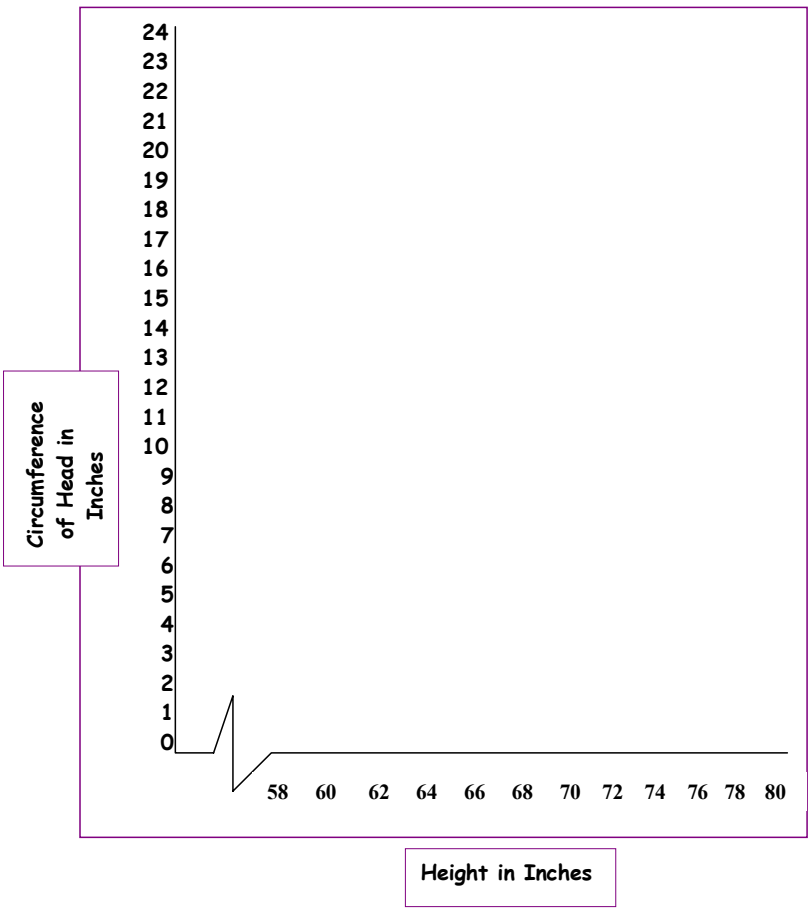
Participant Page

Scatter Plot Activity

You should become familiar with:

- **Scatter plot**
- **Correlation**:

Wall Scatter Plot



1. Place *one* sticky dot on the wall chart that will represent *both* your height in inches and the circumference of your head in inches. (This activity will be discussed later in the lesson)

Instructor Page

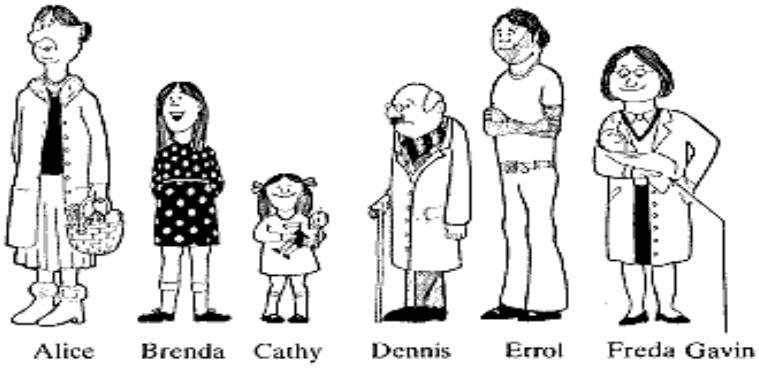
Scatter Plot (continued)

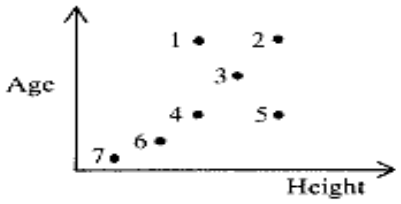
3. **Introduce the idea of Scatter Plots using the following picture:**

A Scatter Plot shows relationships or associations between two sets of data.

1. The Bus Stop Queue

Who is represented by each point on the scattergraph, below?





Permission to use granted from: Swan (1985) The Language of Functions and Graphs, Shell Centre for Mathematical Education, University of Nottingham, England. (http://www.MathShell.com/scp/lfg50.htm)

- **Answers to scatter plot:**

Alice #2
Brenda # 4
Cathy #6
Dennis # 1
Errol # 5
Freda # 3
Gavin # 7

M2T2

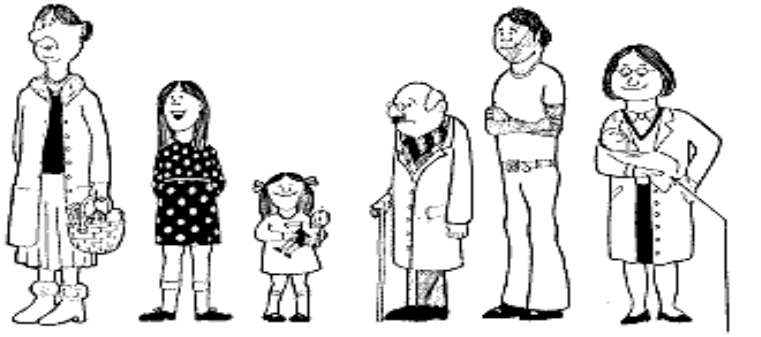
Participant Page

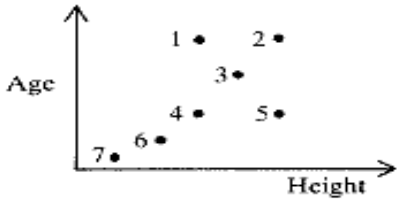
Scatter Plot (continued)

2. **Look at the following picture and determine which dot represents which person.**

1. The Bus Stop Queue

Who is represented by each point on the scattergraph, below?





- Which number correlates with Alice's height and age? _____

Brenda's? _____ Dennis's? _____

Cathy's? _____ Errol's? _____

Freda's _____ Gavin's? _____

Scatter Plot (continued)

Instructor Page

4. **Group Activity:**
   a. **Each participant has placed a numbered sticky dot at the appropriate location on the wall Scatter Plot that represents both his/her circumference of head *and* height in inches.**
   b. **This will provide a visual representation of all participants.**
   c. **Discuss the scale used on the scatter plot. Note that missing data (0-57) is represented by ⌐ (to the right of zero)**

5. **Participants will fill out information chart with help from other participants**
6. **Using information from the charts, participants determine which point identifies which person in class**
7. **Discuss results: When would scatter plots be helpful? (when trying to see a correlation between 2 sets of data)**

Scatter Plot Information Sheet

Name	Circumference of Head	Height in Inches
1..		
2..		
3..		
4.		
5.		
6.		
7.		
8.		
9.		
10.		
11.		
12.		
13.		
14.		
15.		
16.		
17.		
18.		
19.		
20.		
21.		
22.		
23.		
24.		
25.		

M2T2

Participant Page

Scatter Plot (continued)

3. Get the information needed to complete the following chart from other participants.

Scatter Plot Information Sheet

Name	Circumference of Head in Inches	Height in Inches
1..		
2..		
3..		
4.		
5.		
6.		
7.		
8.		
9.		
10.		
11.		
12.		
13.		
14.		
15.		
16.		
17.		
18.		
19.		
20.		
21.		
22.		
23.		
24.		
25.		

4. **Using the information on the chart, match the numbered sticky dot on the scatter plot to the correct participant's name.**
5. **When would using a scatter plot be helpful?**

_____

_____

Box and Whisker Plots

Instructor Page

Minimum is the smallest value in a set of data

Maximum is the largest value in a set of data

1st Quartile is the midpoint of lower half of data.

3rd Quartile is the midpoint of upper half of data

How Many Letters Are in Your Last Name?

A box and whisker plot, also called a box plot, is made up of a rectangle that represents the middle 50% of the data and "whiskers" at each end to represent the upper and lower 25%. Box and whisker plots are helpful in showing the distribution of the data.
This activity provides students with an opportunity to become actively involved in creating a box plot.

Materials: adding machine tape, yarn, unifix cubes, card stock with the following terms printed: *Median, Minimum, Maximum, 1st Quartile, 3rd Quartile*

Guideposts:

Have students take one unifix cube for each letter of their last name.

- Students will make a tower out of the given unifix cubes
- Students will order themselves numerically from smallest to largest number of unifix cubes (They will form a horizontal line)

To determine the **Median**:

If line has even number of participants:

- **One person will step forward from the left end of the line. One person will step forward from the right end of the line. (Continue this process until there are two persons left)**
- **Have median positions hold a card together that says Median (*Median is really an average of these 2 values*)**

If line has odd number of participants:

- **One person will step forward from the left end of the line. One person will step forward from the right end of the line. (Continue this process until there is one persons left)**
- **Have median position hold a card that says Median**

*** All students move back into the position they had in the beginning line***

M2T2

Participant Page

Box and Whisker Plats

How Many Letters Are in Your Last Name?

You should become familiar with the following terms:

- **Minimum**
- **Maximum**
- **1st Quartile**
- **3rd Quartile**

Warm-up Activity:

1. Imagine everyone's last name in the listed in order by number of letters. They will be listed from least to greatest. Where do you think that your name will be in the list?
2. What do you think will be the average number of letters in the class member's last names?
3. Using unifix cubes, how could you demonstrate the length of each name?

Class Activity:

1. Make human Box & Whisker Plot by following teacher instructions.
2. Using the class data, make a bar graph to represent the length of all last names in the class

Post Activity:

1. How close was your prediction where you thought your name would be to the actual place it was?

_____

_____

Using the class data:

1. What name is located at the following places?

Minimum _____

1st Quartile _____

Median _____

Upper Quartile _____

Upper Extreme _____

How Many Letters in Your Last Name? (continued)

Instructor Page

***** All students move back into the position they had in the beginning line*****

- **To show the Minimum give card labeled Minimum to person(s) with the smallest unifix tower (smallest value)**
- **To show the Maximum give card labeled Maximum to person(s) with the highest unifix tower (highest value)**

To determine the **3rd Quartile**

Have all the students who are standing to the *right* of the person(s) holding the median card move forward two steps

If line has even number of participants:

- One person will step forward from the left end of the line. One person will step forward from the right end of the line. (Continue this process until there are two persons left)
- Have these two people hold a card together that says **3rd Quartile**

If line has odd number of participants:

- One person will step forward from the left end of the line. One person will step forward from the right end of the line. (Continue this process until there is one person left)
- Have this person hold a card that says **3rd Quartile**

To determine the **1st Quartile**

Repeat the process used to find *3rd Quartile* but start with the group of students that are to the *left* of the person(s) holding the median card

- Have the final person(s) hold a card that says **1st Quartile**

M2T2

Participant Page

How Many Letters in Your Last Name? (continued)

2. Is there a mode? _____

3. What is the range? _____

4. What two values are needed to determine the range? _____

_____

5. Predict the length of the last name of a new student coming to the classroom next week. _____

6. Why did you choose that number?

_____

_____

7. Using our class data, is it really possible to predict the length of the last name of a new student ? Why or why not

_____

_____

How Many Letters in Your Last Name? (continued)

Instructor Page

To determine **Box Plot** (the middle 50% of the entire line)

- Using adding machine tape, wrap the middle 50% of the group (all students standing between the 1st and 3rd Quartiles)

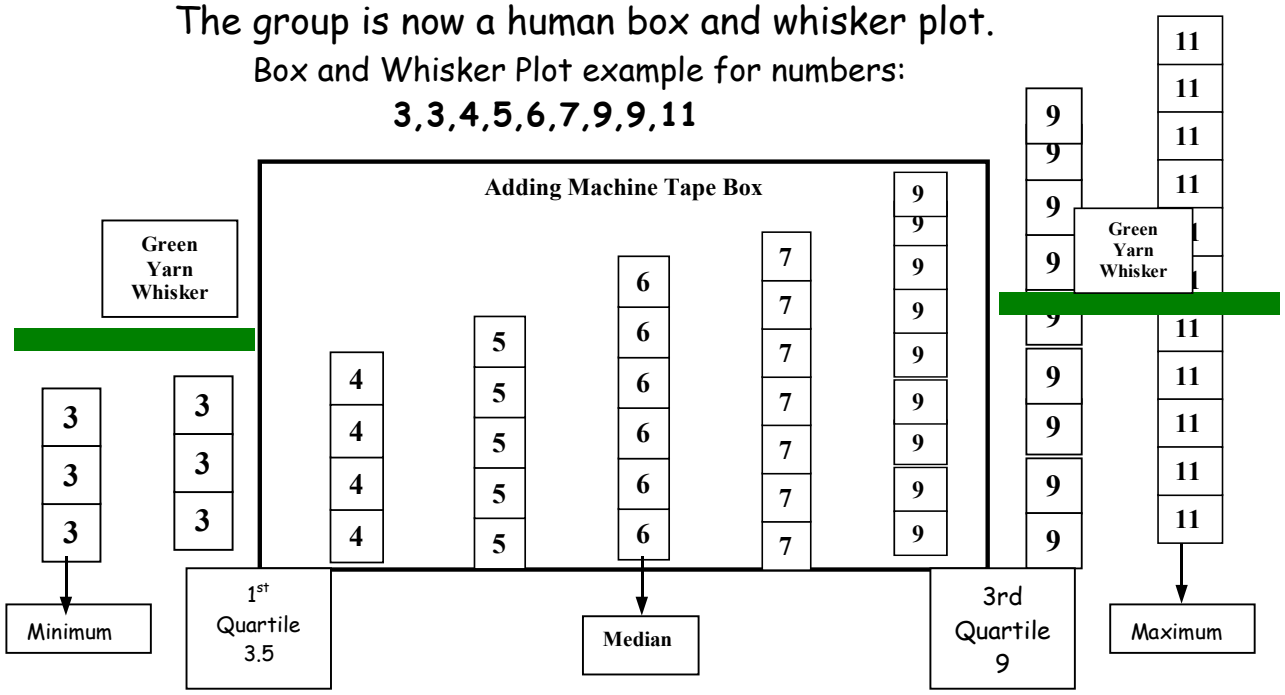
To determine the **whiskers** (distance from each end to 3rd and 1st quartiles)

- Extend yarn from person holding Minimum card to person holding the 1st Quartile card
- Extend yarn from person holding Maximum card to person holding the 3rd Quartile card

The group is now a human box and whisker plot.

Box and Whisker Plot example for numbers:

3,3,4,5,6,7,9,9,11



The example Box and Whisker Plot shows us that:

- **One quarter of the data lies below 3.5**
- **One quarter of the data lies between 3.5 and 6**
- **One quarter of the data lies between 6 and 9**
- **One quarter of the data lies between 9 and 11**

2. Using the class data, have the students make a bar graph that shows the lengths of all last names.

THIS PAGE INTENTIONALLY BLANK

How Many Letters in Your Last Name? (continued)

Instructor Page

The **range and mode** can be determined

- Range is difference between Maximum and Minimum (Example problem = 8)
- Mode is the number(s) that appears most frequently (Example problem = 3 and 9)

To find the **mean** of the data set:

- Student exchange unifix cubes UNTIL all students have the same number of cubes (All towers will be equal in height. Sometimes there might be a few cubes left if there is a remainder)

MEAN from example: (After exchanging cubes, all students will end up with 6 unifix cubes… 3 will be left over because average is 6.333)

Discussion Questions:

1. **Using our class data, is it really possible to predict the length of a new student's last name?**

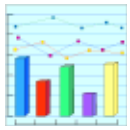
2. **What are the advantages of creating a box and whisker plot?**
   - It visually shows the median.
   - It divides the data into four groups based on size.
   - It shows the spread for a set of data.

3. **A box and whisker plot would serve what other data?**
   - Height in inches
   - Age in months
   - Hours watching TV for one week
   - Etc.

4. **What other type of graph would be appropriate for use with this data?**
   - Bar graph or Histogram

THIS PAGE INTENTIONALLY BLANK

A Graph is Worth a Thousand Words



Instructor Page

Students should be familiar with the various types of graphs.

- **Bar Graphs**
- **Line Graphs**
- **Circle Graphs**
- **Stem and Leaf Plots**
- **A Scatter Plot**

Materials: Student Worksheet

Guidepost:

1. Discuss the following information with class:

Creating graphs is an excellent way to show information. It is important to use an appropriate graph with a given set of data.

- *Bar Graphs, Histograms, and Pictographs* are used to compare countable data or categories among various groups. Usually one axis is numerical and one is not.
  - Bar Graphs show a comparison of data change over a given time
  - Histograms show a comparison of data that falls into different ranges or intervals
- *Line Graphs*
  - Used to indicate change over a period of time
  - Since time is continuous, all data cannot be place on the line graph.
- *Circle Graphs*
  - Used to compare parts as they relate to the whole
  - Used when the whole is divided into different sections, parts, or regions
- *Stem and Leaf Plots, Scatter Plots, Frequency Tables, and Box and Whisker Plots* are all used to show how data is clustered. They are best used when numbers within a single category are to be compared.
  - A Stem and Leaf Plot is a way to show frequency distribution. (median and mode are easy to interpret)
  - A Scatter Plot is a basic graphic tool that illustrates the relationship between two variables. The dot on the scatter plot represents a point that shows the relationship between the two variables.
  - A Box and Whisker Plot is a graph that shows how far apart and how evenly data is distributed in the four quadrants. (The median is easily identified.)
  -

2. Have students fill out student worksheet. Discuss answers.

M2T2

Participant Page

A Graph is Worth a Thousand Words

You should be familiar with the various types of graphs.

- **Bar Graphs**
- **Line Graphs**
- **Circle Graphs**
- **Stem and Leaf Plots**
- **A Scatter Plot**

After listening to a discussion led by the teacher, decide which type of graph could be used to show the following information:

1. What graph could be used to show how the number of farms has changed in Illinois from 1920-2000?

_____ Why did you choose that type of graph?

_____

2. What graph would best show the ages of the school soccer club members?

_____ Why did you choose that type of graph?

_____

3. What graph would best show the favorite brand of shoes for our 5th grade class?

_____ Why did you choose that type of graph?

_____

4. What graph would best show how a 6th grader spends his money during a typical week?

_____ Why did you choose that type of graph?

_____

5. What graph would best show a comparison of the height and shoe size of the students in our 4th grade class?

_____ Why did you choose that type of graph?

_____

THIS PAGE INTENTIONALLY BLANK

M2T2

Participant Page

Appendix A

Tally Sheets for Probability Activities
Coin Activity

Ten Trials: (coin throw)
Mark each trail with a W for a win or an L for a loss
Remember that a player must throw 2 odd numbers to win.

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10
Player 1										
Player 2										

30 Trials: (coin throw)
Mark each trail with W for a win OR an L for a loss
Remember that a player must throw 2 odd numbers to win.

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10
Player 1										
Player 2										

	Trial 11	Trial 12	Trial 13	Trial 14	Trial 15	Trial 16	Trial 17	Trial 18	Trial 19	Trial 20
Player 1										
Player 2										

	Trial 21	Trial 22	Trial 23	Trial 24	Trial 25	Trial 26	Trial 27	Trial 28	Trial 29	Trial 30
Player 1										
Player 2										

THIS PAGE INTENTIONALLY BLANK

M2T2 Participant Page

Appendix B

Tally Sheet for Probability Marble Activity

Game 1: (*1 Red marble and 2 Blue marbles*)
Mark each Trial with a W for a win and an L for a Loss

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10
Player 1										
Player 2										

	Trial 11	Trial 12	Trial 13	Trial 14	Trial 15	Trial 16	Trial 17	Trial 18	Trial 19	Trial 20
Player 1										
Player 2										

	Trial 21	Trial 22	Trial 23	Trial 24	Trial 25	Trial 26	Trial 27	Trial 28	Trial 29	Trial 30
Player 1										
Player 2										

	Trial 31	Trial 32	Trial 33	Trial 34	Trial 35	Trial 36	Trial 37	Trial 38	Trial 39	Trial 40
Player 1										
Player 2										

	Trial 41	Trial 42	Trial 43	Trial 44	Trial 45	Trial 46	Trial 47	Trial 48	Trial 49	Trial 50
Player 1										
Player 2										

Was Game 1 fair? How do you know?

_____

_____

THIS PAGE INTENTIONALLY BLANK

M2T2 Participant Page

Appendix C

Tally Sheet for Probability Marble Activity

Game 2: (*2 Red marble and 2 Blue marbles*)

Mark each Trial with a W for a win and an L for Loss

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10
Player 1										
Player 2										

	Trial 11	Trial 12	Trial 13	Trial 14	Trial 15	Trial 16	Trial 17	Trial 18	Trial 19	Trial 20
Player 1										
Player 2										

	Trial 21	Trial 22	Trial 23	Trial 24	Trial 25	Trial 26	Trial 27	Trial 28	Trial 29	Trial 30
Player 1										
Player 2										

	Trial 31	Trial 32	Trial 33	Trial 34	Trial 35	Trial 36	Trial 37	Trial 38	Trial 39	Trial 40
Player 1										
Player 2										

	Trial 41	Trial 42	Trial 43	Trial 44	Trial 45	Trial 46	Trial 47	Trial 48	Trial 49	Trial 50
Player 1										
Player 2										

Was Game 2 fair? How do you know?

_____

_____

THIS PAGE INTENTIONALLY BLANK

M2T2 Participant Page

Appendix D

Tally Sheet for Probability Marble Activity

Game 3: ***(1 Red marble and 3 Blue marbles)***

Mark each Trial with a W for a win and an L for Loss

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10
Player 1										
Player 2										

	Trial 11	Trial 12	Trial 13	Trial 14	Trial 15	Trial 16	Trial 17	Trial 18	Trial 19	Trial 20
Player 1										
Player 2										

	Trial 21	Trial 22	Trial 23	Trial 24	Trial 25	Trial 26	Trial 27	Trial 28	Trial 29	Trial 30
Player 1										
Player 2										

	Trial 31	Trial 32	Trial 33	Trial 34	Trial 35	Trial 36	Trial 37	Trial 38	Trial 39	Trial 40
Player 1										
Player 2										

	Trial 41	Trial 42	Trial 43	Trial 44	Trial 45	Trial 46	Trial 47	Trial 48	Trial 49	Trial 50
Player 1										
Player 2										

Was Game 3 fair? How do you know?

_____

_____

THIS PAGE INTENTIONALLY BLANK

Participant Page

Appendix E

Student Worksheet
A Graph is Worth a Thousand Words

After listening to a discussion led by the teacher, decide which type of graph could be used to show the following information:

1. **What graph could be used to show how the number of farms has changed in Illinois from 1920-2000?**

_____ Why did you choose that type of graph?

_____

2. **What graph would best show the ages of the school soccer club members?**

_____ Why did you choose that type of graph?

_____

3. **What graph would best show the favorite brand of shoes for our 5th grade class?**

_____ Why did you choose that type of graph?

_____

4. **What graph would best show how a 6th grader spends his money during a typical week?**

_____ Why did you choose that type of graph?

_____

5. **What graph would best show a comparison of the height and shoe size of the students in our 4th grade class?**

_____ Why did you choose that type of graph?

_____

THIS PAGE INTENTIONALLY BLANK

M2T2

Participant Page

Appendix F

Glossary of Terms

Box and Whisker Plot: is good for showing center and spread for a set of data. This plot also groups data into four quartiles.

Correlation: is strength of the relationships of the two variables being measured. Correlation can be positive or negative. Sometimes there is no correlation between the two variables.

Experimental Probability: is based on the results of a series of trials

Fair Game: occurs when all players have an equal chance of winning

1st Quartile: is the midpoint of the lower half of data set.

Histogram: is a bar graph that shows a comparison of data that falls into different ranges or intervals.

Maximum: is the largest value in a set of data.

Mean: is the sum of a set of numbers, divided by the number of numbers in the set. The mean is often called the "average".

Median: is the middle number in a series of numbers where they are arranged in order (least to greatest or greatest to least).

Minimum: is the smallest value in a data set.

Mode: is the number in a set of data that occurs the most often.

Range: is the difference between the largest number in the set and the smallest number in the set.

Sample: is a group of data that represents the entire group's population.

Scatter Plot: is a graph that displays quantitative data as points in two or more dimensions. Scatter plots are used to visualize relationships between variables.

Stem and Leaf Plot: is a way to display data where certain digits are used as "stems" and other as "leaves"

Theoretical Probability: is the number obtained by dividing the number of successful outcomes by the total number of possible outcomes in the sample set.

3rd Quartile: is the midpoint of the upper half of a data set.

Tree Diagram: is a network of points connected by line segments. Tree diagrams can represent probability situations in which there is a series of events.

Variable: is a quantity that varies such as the number of hours of television watched each week.

Email questions and comments to
m2t2@mail.mste.uiuc.edu

Statistics